...SITY
...NIVERSITEIT
BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

[redacted]

SOME MODIFICATIONS AND APPLICATIONS OF RUBINSTEIN'S PERFECT EQUILIBRIUM MODEL OF BARGAINING

G.J.M. van den Boom

FEW 259

Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining

G.J.M. van den Boom*

I owe thanks to Dr.A.J. de Zeeuw and Dr. J.E.J.Plasmans for their indispensable help and valuable comments during my work on this paper.

Contents

1. Introduction

More and more economists feel that game-theoretic analysis may help to better understand a variety of economic processes. Examples of theoretical and applied work in this area are: Kreps and Wilson (1982 b) on oligopolistic competition; Shaked and Sutton (1984) on wage bargaining; de Zeeuw (1984) on policy design; Grossman and Richardson (1985) on international trade; Withagen (1984) on exhaustible resources; Binmore and Herrero (1985) on price mechanisms; Kooreman and Kapteyn (1985) on labor supply decisions.

It is often observed that the parties involved (i.e. the players in the game) have conflicting objectives and tend not to cooperate. Furthermore, when promisses are not binding (i.e. no commitments can be made), players tend not to believe each others announcements, unless credibility is beyond dispute.

The theory of non-cooperative dynamic games forms a natural framework to describe behaviour in the processes above (see Basar and Olsder (1982)). The most important equilibrium concept in non-cooperative games is due to Nash (1951) and has become known as Nash equilibrium (NE). In a NE the strategies are such that none of the players can improve upon the outcome of the game, given the strategies of his opponents; thus in a NE no one can achieve a better pay-off as long as the others do not deviate from their equilibrium strategies. An important property of the NE concept is time-consistency <u>on</u> the equilibrium path; that is: reoptimisation does not change the planned actions for the remainder of the game as long as everyone plays his equilibrium strategy.

A typical feature of most bargaining situations is that threats (defined as announcements on future moves in the game) are not binding. This implies that the players can reoptimize at any decision node, whereever the game has evolved to thus far. If, - faced with the developments thus far as a fait accompli -, it is not rational to execute a threat, the threat is called incredible. Strategies in a NE may contain such incredible threats; in other words: the NE concept cannot assure time-consistency <u>off</u> the equilibrium path (see Meijdam and de Zeeuw (1986) for a discussion of time-consistency in dynamic game theory).

This drawback of the NE concept lead Selten (1975) to refine NE and to introduce the concept of subgame perfect equilibrium (PE). Subgame perfectness is defined as time-consistency <u>off</u> the equilibrium path. The PE concept prescribes that every player chooses a Nash-strategy for the remainder of the game, for every informationset that can be reached (including the ones that will not be reached in equilibrium). That is: no player can unilaterally improve upon his pay-off by reoptimisation, whatever has happened so far. When PE strategies are played, the corresponding outcome of the game is self-enforcing and the threats involved are credible.

This paper is concerned with the implementation of this PE concept into bargaining theory. A seminal contribution is Rubinstein (1982), whose analysis of a two player bargaining model serves as the point of departure.

Section 2 describes the main features of the basic model. Two players (e.g. worker and firm, union and employer's organisation or government and private sector) have to reach an agreement on the partition of a pie (e.g. a wage increase or the benefits of economic growth). On turn each player makes a proposal and his opponent either accepts or rejects.
In case of acceptance the bargaining ends and in case of rejection the bargaining continues with a proposal by the opponent in the next round, etcetera. In this model the NE concept cannot predict an outcome; in fact it will be shown that any partition is the outcome of some NE strategies. Moreover, - and this is worse -, NE strategies generally contain incredible threats and one of the players can anticipate on this and obtain a better expected pay-off by deviating from his NE strategy. In contrast the PE concept is able to predict self-enforcing outcomes as the result of PE (subgame perfect) strategies. Mostly, the set of PE outcomes consists of only one element: the unique PE outcome of the bargaining game. These result have been proved in Rubinstein (1982) and are reconstructed in this paper using arguments presented in Shaked and Sutton (1984) and Sutton

(1986). At the end of section 2 it is shown that risk aversion is a disadvantage in the model (see Roth (1985)).

Section 3 deals with asymmetries in the bargaining procedure. One of the players (i.c. the firm) is allowed to switch to an other opponent if no agreement is reached after some minimum number of bargaining rounds. The effects of such an outside option is analysed in Shaked and Sutton (1984). Their main result is generalized: firstly a broader class of preferences is considered (including the case of different discountfactors and the case of fixed bargaining costs per round); secondly the players may have a different reactiontime[1] and finally not only the PE outcome for the original game is given, but also the PE outcomes for every subgame (if no agreement has been reached yet, for whatever reason).

Section 4 analyses incomplete information.[2] Following Rubinstein (1985a) it is assumed that one of the players (i.c. the worker) is unsure about his opponent's time preferences: either he is strong (i.e. the patient type) or he is weak (i.e. the impatient type). It is shown that if the opponent's type is weak, then he is better off in the incomplete information case (bluffing!) and if he is strong, then he is better off in the complete information game. More specifically: above some critical reputation of being strong, the opponent (whether weak or strong) will get the same equilibrium outcome he would get when he was known to be strong. Below that reputation his equilibrium pay-off is worse, but always at least what he would get when he was known to be weak. An interesting result in this incomplete information case is further that equilibrium may involve periods of disagreement (which is irrational in the complete information case).

Section 5 is an application of the basic model to optimal control in a policy-model (see Stefanski and Cichocki (1986)).

Section 6 summarizes and concludes.

2. Bilateral Monopoly

2.1 The Basic Bargaining Model

Consider the following problem:
A firm wants to hire a worker, whose reservation wage equals zero and whose (normalised) gross labor value is to be divided between wage w and profit (1-w), $w \in [0,1]$. What strategies will the firm and the worker adopt in trying to reach an agreement? What agreement(s) will emerge as a result of the strategies chosen?

To answer these questions, which are typical for the so-called strategic approach[3], it is useful to construct a game theoretic bargaining model. The bargaining process is represented as a non-cooperative dynamic game, in which the players move sequentially. None of the players has an outside option, information is complete and the bargaining procedure is as follows (see Rubinstein (1982)):

Figure 1
Game 1 Bilateral Monopoly

t	ϑ	worker's move		firms's move
0	a			w_0
	b	A_0	$\longleftarrow$	
1	a	w_1		
	b		$\longrightarrow$	A_1
2	a			w_2
	b	A_2	$\longleftarrow$	
3	a	w_3		
	b		$\longrightarrow$	A_3
4	a			w_4
	b	A_4	$\longleftarrow$	
.	.	.		.
.	.	.		.
.	.	.		.
.	.	.		.

In the first bargaining round ($t=0$) the firm proposes some wage $w_0 \in [0,1]$ to the worker at stage $\vartheta = a$. If the worker accepts by playing $A_0 = Y$ at stage $\vartheta = b$, the game ends and the outcome is $(w,t) = (w_0,0)$. If, however, the worker rejects w_0 (i.e. $A_0 = N$) then he can make a counteroffer w_1 in the second bargaining round $t=1$ at stage $\vartheta = a$. Now it is the firm's turn either to accept or to reject this w_1, at stage $\vartheta = b$, the first move leading to the outcome $(w,t) = (w_1,1)$ and the latter to another proposol, w_2, in the third round, etcetera.

Note that the original game reappears every two rounds.

To describe (equilibrium) strategies in game 1, some straightforward notation is introduced.

$f := (A_0, w_1, A_2, w_3, \dots)$	: worker's strategy (player 1)
$g := (w_0, A_1, w_2, A_3, \dots)$	: firm's strategy (player 2)
$t \in \{0, 1, 2, 3, \dots\}$	: bargaining rounds, discrete time
$\vartheta \in \{a, b\}$	: subdivision of a round t; at stage a the wage w_t is proposed and at the subsequent stage b the reaction A_t on w_t is given.
$w_t \in [0,1]$	: wage proposal at t
$A_t \in \{Y, N\}$	: reaction on w_t: Y means "accept w_t" and N means "reject w_t"
$P(f, g) \in [0,1] \times \{0, 1, 2, 3, \dots\}$	: bargaining outcome when the workers plays f and the firm plays g. If agreement is ever reached (i.e. $A_t = N, \forall_t$), then assign $P(f, g) = (w,\infty)$ for some $w \in [0,1]$ to denote perpetual disagreement.
$\succsim_i$	: complete, reflexive and transitive preference relation of player i over the set of ordered pairs $P(f,g) \in [0,1] \times \{0, 1, 2, \dots\}$; $i = 1, 2$.

2.2 Nash Equilibrium

Firstly the conventional Nash equilibrium concept in the game 1 is defined (see Nash (1951)).

Definition 1 (Nash equilibrium; rationality ex ante).
A pair of strategies (f^*, g^*) is called a Nash equilibrium (NE) if
A) $P(f^*, g^*) \succsim_1 P(f, g^*)$, $\forall f$
B) $P(f^*, g^*) \succsim_2 P(f^*, g)$, $\forall g$

The definition requires that none of the players has an incentive to deviate from this strategy, given the strategy of his opponent.
Two things are not attractive in the NE concept here. Firstly, it cannot predict certain outcomes of the game (see proposition 1 below) and secondly, many NE strategies are not time-consisten off the equilibrium path (i.e. not subgame perfect, see example 1 below).

Let time be valuable for both, wage desirable for the worker and profit for the firm.[4] Then:

Proposition 1 (weakness NE)
(f^*,g^*) NE such that $P(f^*,g^*) = (w,t) \Leftrightarrow (w,t) \succsim_1 (0,0)$
$(w,t) \succsim_2 (1,0)$

Proof
($\Rightarrow$)To see that the conditions are necessary, suppose one of the conditions is not satisfied. Then (f^*,g^*) cannot be a NE: if $(0,0) \succ_1 (w,t)$ then the worker can better change his strategy and accept anything at t=0; also if $(1,0) \succ_2 (w,t)$ then the firm can better change its strategy and propose
$w_o = 1$ at $t = 0$.
($\Leftarrow$) To prove sufficiency, suppose for some $w \in [0,1]$ and some $t \in \{0, 1, 2, ...\}$ $(w,t) \succsim (0,0)$ and $(w,t) \succsim_2 (1,0)$.
A NE (f^*,g^*) leading to the outcome $P(f^*,g^*) = (w,t)$ can be constructed as follows:

until t is reached, the firm constantly proposes full profit (i.e. $w_s = 0$, $s = 0, 2, \dots, \tau_0$ with $\tau_0 = t-2$ if t even and $\tau_0 = t-1$ if t odd) and rejects anything less (i.e. $A_s = N$ if $w_s > 0$; $A_s = Y$ if $w_s = 0$, $s = 1, 3, \dots . \tau_1$ with $\tau_1 = t-2$ if t odd and $\tau_1 = t-1$ if t even). Also the worker constantly proposes full wage (i.e. $w_s = 1$, $s = 1, 3, \dots, \tau_1$) and rejects anything less (i.e. $A_s = N$ if $w_s < 1$; $A_s = Y$ if $w_s = 1$, $s = 0, 2, \dots, \tau_0$). From the period t onwards, the firm as well as the worker propose w (i.e. $w_s = w$, $s = t, t+1, \dots$) and the worker rejects any smaller wage (i.e. $A_s = N$ if $w_s < w$; $A_s = Y$ if $w_s \geq w$, $s = \tau_0 + 4, \dots$) and the firm rejects any larger wage (i.e. $A_s = N$ if $w_s > w$; $A_s = Y$ if $w_s \leq w$, $s = \tau_1 + 2, \tau_1 + 4, \dots$).

If for example t is even, then we construct ($\tau_0 = t-2$; $\tau_1 = t-1$):

$$f^* = (A_0, 1, A_2, 1, \dots , A_{\tau_0}, 1, A_{\tau_0+2}, w, A_{\tau_0+4}, \dots)$$

$$g^* = (0, A_1, 0, A_3, \dots , 0, A_{\tau_1}, w, A_{\tau_1+2}, w, \dots)$$

For t is odd we have ($\tau_0 = t-1$; $\tau_1 = t-2$):

$$f^* = (A_0, 1, A_2, 1, \dots , 1, A_{\tau_0}, w, A_{\tau_0+2}, w, A_{\tau_0+4}, w, \dots)$$

$$g^* = (0, A_1, 0, A_3, \dots , A_{\tau_1}, 0, A_{\tau_1+2}, w, A_{\tau_1+4}, w, \dots)$$

By construction of (f^*,g^*),$P(f^*,g^*) = (w,t)$. To see that (f^*,g^*) is a NE we have to check:

$$(w,t) \succsim_1 P(f,g^*) \quad , \forall f$$

$$(w,t) \succsim_2 P(f^*,g) \quad , \forall g$$

Given g^* the worker cannot improve upon the outcome by changing f^*, because either he can get $P(f,g^*) = (0,s)$ for some $s \in \{0, 1, 2, \dots , t-1\}$ or he can get $P(f,g^*) = (w,s)$ for some $s \in \{t+1, t+2, \dots \}$. In both cases (w,t) is preferred: $(w,t) \succsim_1 (0,0) \succsim_1 (0,s)$, $s < t$ respectively $(w,t) \succsim_1 (w,s)$, $s \geq t$.

Also for the firm, given f^*, the only alternative outcomes that can be reached are $P(f^*,g) = (1,s)$ for some $s < t$ or $P(f^*,g) = (w,s)$ for some $s > t$. In both cases (w,t) is preferred: $(w,t) \succsim_2 (1,0) \succsim_2 (1,s)$, $s < t$ respectively $(w,t) \succsim_2 (w,s)$, $s \geq t$.
Thus (f^*,g^*) is a NE. □

Note that proposition 1 implies that in the first round ($t=0$) any wage $w \in [0,1]$ can be the result of a NE. Moreover, a NE wage $w=0$ or $w=1$ can only occur in the first bargaining round; NE wages between 0 and 1 may occur in some future bargaining round t (if $(w,t) \succsim_1 (0,0) \wedge (w,t) \succsim_2 (1,0)$).

The following example illustrates the (well known) fact that Nash equilibria can be time inconsistent off the equilibrium path (subgame imperfectness).

<u>Example</u> (time-inconsistency NE off the equilibrium path).
Consider the utility functions:

$$u_1(w,t) = \delta_1^{\,t}\, w \quad : \text{discounted wage}$$

$$u_2(w,t) = \delta_2^{\,t}\,(1-w) : \text{discounted profit}$$

and let $\delta_1 = 0.4$ and $\delta_2 = 0.9$ be the discountfactors.[5]
A NE leading to the outcome $P(f^*,g^*) = (0.5,\ 0)$ is:

$$f^* = (A_0,\ 0.5,\ A_2,\ 0.5,\ \ldots\ )$$

$$g^* = (0.5,\ A_1,\ 0.5,\ A_3,\ \ldots\ )$$

$$\begin{aligned} \text{with } A_s &= Y \text{ if } w_s \geq 0.5 \\ &= N \text{ if } w_s < 0.5,\ s = 0,\ 2,\ 4,\ \ldots \\ A_s &= Y \text{ if } w_s \leq 0.5 \\ &= N \text{ if } w_s > 0.5,\ s = 1,\ 3,\ 5,\ \ldots \end{aligned}$$

Although (f^*, g^*) is a NE, it is easily seen that these strategies are not likely to be played if no commitment is made with respect to announced behaviour.

Suppose for example that the firm proposes $w_0 = 0.4$ in the first round. Following f^* the worker rejects w_0 and the outcome for the remainder of the game is (0.5, 1). But if the worker expects this to happen, then, - faced with $w_0 = 0.4$ as a fait accompli -, he can better accept w_0, because $u_1(0.4,0) = 0.4 > 0.2 = u_1(0.5,1)$. Thus the threat of the worker to reject any wage less than 0.5 in the first round is incredible to a forward looking firm, which expects the worker to reoptimise at each decision node. In fact it will be shown below (corollary 1) that the firm can propose a wage w_0 as low as $y = \frac{2}{37}$ and it is still rational for the worker to accept this w_0.

2.3 Perfect equilibrium

To reduce the set of equilibrium wages and to avoid time inconsistent behaviour of the equilibrium path the concept of Perfect equilibrium (PE) is a very useful refinement of the NE concept. The PE concept (for dynamic, extensive form games) is due to Selten (1975). To define PE subgame-strategies are needed:

$f \mid t_\vartheta$: worker's subgamestrategy after the game has evolved just up to stage t_ϑ; $t \in \{0, 1, 2, \dots\}$
$\vartheta \in \{a,b\}$

$g \mid t_\vartheta$: idem firm.

From figure 1 it is easily seen that:

$f = (A_0, w_1, A_2, w_3, \dots)$ $\qquad$ $g = (w_0, A_1, w_2, A_3, \dots)$

$f|0a = f$ $\qquad$ $g|0a = (w_0, A_1, w_2, \dots)$

$f|0b = f$ $\qquad$ $g|0b = (A_1, w_2, A_3, \dots)$

$f|1a = (w_1, A_2, w_3, \ldots)$ $\qquad$ $g|1a = g|0b$

$f|1b = (A_2, w_3, A_4)$ $\qquad$ $g|1b = g|0b$

$f|2a = f|1b$ $\qquad$ $g|2a = (w_2, A_3, w_4, \ldots)$

$f|2b = f|1b$ $\qquad$ $g|2b = (A_3, w_4, A_5, \ldots)$

$f|3a = (w_3, A_4, w_5, \ldots)$ $\qquad$ $g|3a = g|2b$

$f|3b = (A_4, w_5, A_6, \ldots)$ $\qquad$ $g|3b = g|2b$ $\qquad$ etc.

<u>Definition 2</u> (Perfect equilibrium; Bilateral Monopoly; rationality in all possible subgames)

A pair of strategies (f^*, g^*) is called a Perfect equilibrium (PE) if $\forall t \in \{0, 1, 2, \ldots\}$, $\forall \vartheta \in \{a,b\}$:

A) $P(f^*|t_\vartheta, g^*|t_\vartheta) \underset{\sim}{>}_1 P(f|t_\vartheta, g^*|t_\vartheta)$, $\forall f|t_\vartheta$

B) $P(f^*|t_\vartheta, g^*|t_\vartheta) \underset{\sim}{>}_2 P(f^*|t_\vartheta, g|t_\vartheta)$, $\forall g|t_\vartheta$

In order to get a set of requirements that is easier to handle, Rubinstein (1982) uses an alternative definition.

<u>Reformulation definition 2</u>

Consider the bargaining game 1, figure 1 and let $f|t := f|t_a$ and $g|t := f|t_a$ be the subgamestrategies at the begin of round t (as defined above). The PE conditions A and B are equivalent with:

A) if t odd (i.e. the worker is to make the proposal w_t):

(i) $P(f^*|t, g^*|t) \underset{\sim}{>}_1 P(f|t, g^*|t)$, $\forall f|t$

(ii) $A_{t-1} = Y \Rightarrow (w_{t-1}, t-1) \underset{\sim}{>}_1 P(f|t, g^*|t)$, $\forall f|t$

(iii) $A_{t-1} = N \Rightarrow P(f^{*}|t,\ g^{*}|t) \underset{\sim}{>}_1 (w_{t-1},\ t-1)$

B) if t even (i.e. the firm is to make the proposal w_t):

(i) $P(f^{*}|t,\ g^{*}|t) \underset{\sim}{>}_2 P(f^{*}|t,\ g|t),\ \forall g|t$

(ii) $A_{t-1} = Y \Rightarrow (w_{t-1},\ t-1) \underset{\sim}{>}_2 P(f^{*}|t,\ g|t),\ \forall g|t$

(iii) $A_{t-1} = N \Rightarrow P(f^{*}|t,\ g^{*}|t) \underset{\sim}{>}_2 (w_{t-1},\ t-1)$

Proof

Part A of this reformulation prescribes rational Nash behaviour for the worker in every stage of the game: (i) ensures each wage proposal he makes is rational (i.e. Nash behaviour at the begin of each bargaining round) and (ii)/(iii) ensure every reaction is rational (i.e. Nash behaviour within any round, after being faced with a certain proposal as a fait accompli).

Similarly part B prescribes rationality for the firm in every decision node, after every history. □

To construct PE strategies and to characterize PE outcomes the following functions are essential:

$$d_i \quad : [0,1] \longrightarrow [0,1] \quad ,\ i = 1,\ 2.$$

$$d_1(x) := \min_{(w,0) \underset{\sim}{>}_1 (x,1)} w$$

$$d_2(y) := \max_{(w,1) \underset{\sim}{>} (y,2)} w$$

$d_1(x)$ gives the minimum wage the worker will accept now, given the wage-outcome x in the next round; $1-d_2(y)$ gives the minimum profit the firm will accept now, given the profit-outcome 1-y in the next round.

Following Rubinstein (1982) three additional assumptions on the player's preferences are made[6]:

continuity:

$$\forall s,\ t \in \{0,\ 1,\ 2,\ \ldots\}\quad \forall w^j,\ \bar{w},\ w \in [0,1]$$
$$\left[w^j \longrightarrow \bar{w} \wedge (w^j, s) \underset{\sim i}{>} (w,t),\ \forall j \Rightarrow (\bar{w}, s) \underset{\sim i}{>} (w,t)\right]$$

stationarity:

$$\forall s,\ t \in \{0,\ 1,\ 2,\ \ldots\}\quad \forall w^1,\ w^2 \in [0,1]$$
$$\left[(w^1, s) \underset{\sim i}{>} (w^2,\ s+1) \Leftrightarrow (w^1, t) \underset{\sim i}{>} (w^2,\ t+1)\right]$$

increasing compensation

$$\left[\varepsilon_1 \text{ such that } (w,t) \sim_1 (w + \varepsilon_1(w),\ t+1) \Rightarrow \varepsilon_1 \text{ increasing}\right]$$
$$\left[\varepsilon_2 \text{ such that } (w,t) \sim_2 (w + \varepsilon_2(w),\ t+1) \Rightarrow \varepsilon_2 \text{ decreasing}\right]$$

The interpretation of continuity and stationarity is straightforward. Increasing compensation means that the worker (resp. the firm) suffers more from a delay (i.e. needs more compensation in the future) the higher the postponed wage (resp. profit) is.

Under these assumptions the results in Rubinstein (1982) can be summarised as follows:

Proposition 2 (PE in game 1, complete information)

A) (i) $y \in [0,1]$ is a PE wage proposal by the firm $\Leftrightarrow y = d_1(d_2(y))$

(ii) $x \in [0,1]$ is a PE wage proposal by the worker $\Leftrightarrow x = d_2(d_1(x))$

B) At least one wage $y \in [0,1]$ satisfies $y = d_1(d_2(y))$ and also at least one wage $x \in [0,1]$ satisfies $x = d_2(d_1(x))$.

C) $t \in \{0,\ 2,\ 4,\ \ldots\}$:

(w,t) resp. $(w,t+1)$ is a PE outcome of the subgame starting in round t resp. $t+1 \Leftrightarrow w = y$ resp. $w = x$.

Proof

The proof of part B is straightforward: from the continuity of preferences, d_1 and d_2 and thus $D_{12} := d_1 \circ d_2$ and $D_{21} := d_2 \circ d_1$ are continuous. The compactness of $[0,1]$ ensures D_{12} and D_{2}, have at least one fixed point y respectively x.

The proof of part A and C resembles an idea presented in Shaked and Sutton (1984) and Sutton (1986). For the original, mathematically more elaborated proof see Rubinstein (1982).

(⇐) For y and x satisfying the righthandside conditions $y = d_1(d_2(y))$ and $x = d_2(d_1(x))$, consider the following pair of strategies:

$$f^* = (A_0, x, A_2, x, \ldots\ldots)$$

$$g^* = (y, A_1, y, A_3 \ldots)$$

with

$$\begin{aligned} A_t &= Y \text{ if } w_t \geq y \\ &= N \text{ if } w_t < y \\ A_{t+1} &= Y \text{ if } w_{t+1} \leq x \\ &= N \text{ if } w_{t+1} > x \quad , t = 0, 2, 4, \ldots \end{aligned}$$

It is easily checked that part A of reformulation of PE holds for $t=o$ and part B for $t=1$. This is sufficiënt for (f^*, g^*) to be a PE, because after two rounds the original game reappears and the preferences are stationary.

(⇒) Now it has to be proved that $y = d_1(d_2(y))$ holds for every PE wage proposal by the firm and $x = d_2(d_1(x))$ for every PE proposal by the worker. Let y be a PE wage proposal by the firm, reached in some bargaining round t even. Consider the equilibrium moves at t, $t + 1$, $t + 2$:

time	f^*	g^*
t	$A_t=Y$	$w_t=y$
$t+1$	w_{t+1}	A_{t+1}
$t+2$	A_{t+2}	w_{t+2}

By definition of PE, y must be a PE wage of the subgame starting at t. Moreover, - because every two rounds the original game reappears -, y must be a PE wage of the subgame starting at $t + 2$.

Now let W_1 be the maximum wage (i.e. $1-W_1$ is the minimum profit) the firm is willing to accept in the subgame starting at t + 1. The firm's rational move at t + 1 is only to accept w_{t+1} if w_{t+1} pays at least what y pays at t + 2. Thus the maximum is:

$$W_1 = \max_{(w_{t+1},t+1) \succsim_2 (y,t+2)} w_{t+1} \qquad =: d_2(y)$$

Also in the subgame starting at t a proposal w_t is only acceptable to the worker if w_t at t pays at least what he can expect to attain maximally at t + 1, i.c. W_1. Because (y,t) is a PE outcome, the firm's proposal y shall equal this minimum acceptable wage:

$$y = \min_{(w_t,t) \succsim_1 (w_1,\ t+1)} w_t \qquad =: d_1(W_1)$$

Substitution leads to the required result: $y = d_1(d_2(y)$. Similarly it can be derived that $x = d_2(d_1(x))$ holds for any PE wage proposal by the worker, reached in same bargaining round t + 1 odd:

time	f^*	g^*
t+1	$w_{t+1}=x$	$A_{t+1}=Y$
t+2	A_{t+2}	w_{t+2}
t+3	w_{t+3}	A_{t+3}

Again x is also a PE wage of the subgames starting at t + 1 and t + 3. For the minimum wage the worker is willing to accept at t + 2 (denoted by W_2) the following equality holds:

$$W_2 = \min_{(w_{t+2},\ t+2) \succsim_1 (x,\ t+3)} w_{t+2} \qquad =: d_1(x)$$

Also in the subgame starting at t + 1, a proposal w_{t+1} is only acceptable to the firm if the profit $1-w_{t+1}$ is at least as valuable as the maximum attainable profit at t + 2, i.c. $1-W_2$. Because (x, t+1) is a PE outcome of that subgame, the worker's proposal shall equal this minimum acceptable profit:

$$1-x = \min_{(w_{t+1},\ t+1)\ \gtrsim_2\ (w_2,\ t+2)} (1-w_{t+1})$$

In other words:

$$x = \max_{(w_{t+1},\ t+1)\ \gtrsim_2\ (w_2,\ t+1)} w_{t+1} =: d_2(w_2)$$

Thus $x = d_2(d_1(x))$.
Part C of the proposition is evident from the construction of PE strategies above. □

<u>corollary 1</u> (fixed discounting factors[7])
Let $u_1(w,t) := \delta_1^{\ t}\ w$
and $u_2(w,t) := \delta_2(1-w)$, $\delta_1\ \delta_2 \neq 1$
represent the worker's and the firm's preferences (i.e. discounted wage respectively discounted profit). Then the bargaining game 1 has unique PE wages:

$$y = \frac{\delta_1(1-\delta_2)}{1-\delta_1\ \delta_2} \text{ is the unique PE wage proposal by the firm}$$

$$x = \frac{1-\delta_2}{1-\delta_1\ \delta_2} \text{ is the unique PE wage proposal by the worker}$$

<u>Proof</u>
In this case $d_1(x) = \min_{w \geq \delta_1 x} w = \delta_1\ x$
and $d_2(y) = \max_{1-w \geq \delta_2(1-y)} w = 1 - \delta_2 + \delta_2 y$
Solving for $y = d_1(d_2(y))$ and $x = d_2(d_1(x))$ and applying proposition 2 gives the result □

Notice that the unique outcome of the game 1 (starting at t=0) is (y,0).
Example, continued (time consistency PE off the eq. path). For $\delta_1 = 0.4$ and $\delta_2 = 0.9$ the only PE wage the firm can propose is $y = \frac{2}{37}$ and the only PE wage the worker can propose is $x = \frac{5}{37}$. It is rational for the opponent to accept this and to reject anything that gives him less (see PE strategies in proof of proposition 2, part B($\Leftarrow$)).
Note that the player whose turn it is to make a proposal can use the fact that his opponent is impatient to achieve a better result; it is an advantage to be the one making an offer (i.c. 1-y > 1-x and x > y).

2.4 Risk aversion

The role of risk aversion in the bargaining game 1 is studied in Roth (1985). Risk aversion is reflected within a bargaining round as a concave transformation over the utilities[8].

Risk neutral:

$$u_1(w,t) = \delta_1^{\ t}\ w$$

$$u_2(w,t) = \delta_2^{\ t}\ (1-w)$$

Risk averse:

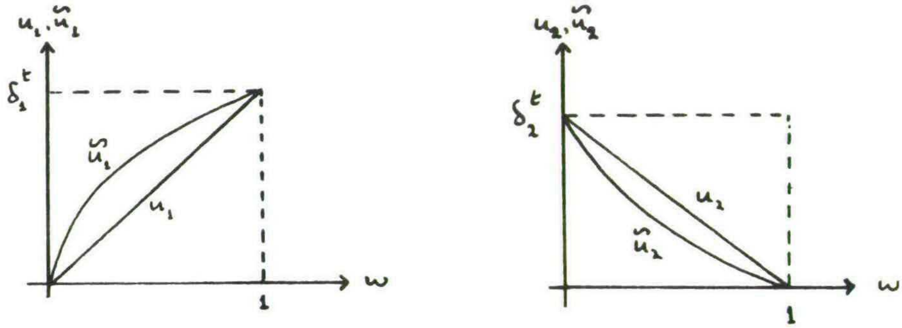
$$\tilde{u}_1(w,t) = \delta_1^{\ t}\ h_1(w)$$

$$\tilde{u}_2(w,t) = \delta_2^{\ t}(1-h_2(w))$$

with $h_1(0) = h_2(0) = 0$
$h_2(1) = h_2(1) = 1$
h_1 continuous, increasing and concave
h_2 " , decreasing and convex

Figure 2

Risk aversion in bargaining round t



To see that risk aversion is a disadvantage in the game 1 we define as before:

$$d_1(x) := \min_{u_1(w,0)\,\geq\, u_1(x,1)} w \quad = \delta_1 x$$

$$\tilde{d}_1(x) := \min_{\tilde{u}_1(w,0)\,\geq\, \tilde{u}_1(x,1)} w \quad = h_1^{-1}(\delta_1 h(x))$$

$$d_2(y) := \max_{u_2(w,0)\,\geq\, u_2(y,1)} w \quad = 1 - \delta_2 + \delta_2 y$$

$$\tilde{d}_2(y) := \max_{\tilde{u}_2(w,0)\,\geq\, \tilde{u}_2(y,1)} w \quad = h_2^{-1}(1-\delta_2 + \delta_2 h_2(y))$$

With these functions and proposition 3 Roth (1985) proved:

Proposition 3 (risk aversion is disadvantageous).
Let x_I and y_I be PE wage proposals in the game 1 between a risk-neutral worker and a risk-neutral firm. Let further x_{II}, y_{II} and x_{III}, y_{III} be PE proposals in the game 1 between a risk-averse worker and a risk-neutral firm respectively between a risk-neutral worker and a risk-averse firm.

Then:

A) $x_{II} \le x_I \le x_{III}$

B) $y_{II} \le y_I \le y_{III}$

Proof

The proof is only given for the utility-functions given above. Using the same arguments the more general case with $u_1(w,t) = \beta_1^{\ t}\ v_1(w)$ and $u_2(w,t) = \beta_2^{\ t}\ v_2(w)$ can be treated (see Roth (1985) and note 7).

Define $D_{21}(x) := d_2(d_1(x))$; $D_{12}(y) := d_1(d_2(y))$;

$$\tilde{D}_{21}(x) := d_2(\tilde{d}_1(x)); \ \tilde{D}_{12}(y) := \tilde{d}_1(d_2(y)).$$

From proposition 3 it follows that x_I is a fixed point of D_{21}; y_I is a fixed point of D_{12}; x_{II} is a fixed point of $\tilde{D}_{21}$ and y_{II} is a fixed point of $\tilde{D}_{12}$.

Because of the concavity of h_1, $h_1(\delta_1 w) \ge \delta_1 h_1(w)$, $\forall w$. Using the monotonicity this implies:

$$\delta_1 x = h_1^{-1}(h_1(\delta_1 x) \ge h_1^{-1}(\delta_1 h_1(x))$$

Substitution gives:

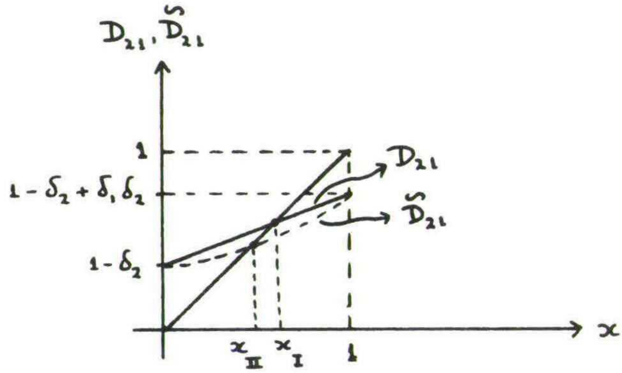
$$\tilde{D}_{21}(x) = d_2(\tilde{d}_1(x)) = 1 - \delta_2 + \delta_2\ h_1^{-1}(\delta_1 h(x))$$

$$\le 1 - \delta_2 + \delta_2 \delta_1 x = D_{21}(x)$$

Thus $x_{II} \le x_I$

Figure 3

Illustration of the proof



Also:

$$\tilde{D}_{12}(y) = h_1^{-1}(\delta_1 h(1 - \delta_2 + \delta_2 y)) \le \delta_1 \; h_1^{-1}(h(1-\delta_2 + \delta_2 y))$$

$$= \delta_1(1 - \delta_2 + \delta_2 y) = D_{12}(y)$$

Thus $y_{II} \le y_I$

The proof of $x_{III} \ge x_I$ and $y_{III} \ge y_I$ is similar

□

Proposition 3 implies for example that the equilibrium wage for a risk-averse worker is lower than the equilibrium wage he would get if he were risk-neutral. Also the firm the equilibrium profit is lower if it is risk-averse.

3 Potential Outsiders

3.1 The Insider-Outsiders Model

Following Shaked and Sutton (1984) an outside option is made available to the firm after having bargained some minimum number of $T \geq 1$ rounds with the same worker (the insider). If no agreement is reached after T rounds, the firm is allowed to switch to an other worker (an outsider). After a switch (S_t = Y at stage ϑ = c) the firm again has to bargain at least T rounds with the new insider and is allowed to switch afterwards if no agreement is reached yet. If the firm does not switch (S_t = N), it can switch again after having rejected the next proposal of the current insider.

As soon as either the insider or the firm accepts a proposal, the bargaining game ends.

Figure 2 gives the bargaining proces described above. It is assumed that T is even. As can be seen this assumption is not restrictive (the procedure is such that the case for $T = t_e$ even is identical to the case for $T = t_e-1$ odd).

At $t = K_1 := 0$ worker 1 enters the game 2 and bargaining can continue until the firm decides the switch after $T_1 \geq T$ rounds. Then at $t = K_2 := T_1$ worker 2 enters. As long as no agreement is reached this continues and at $t = K_j := \sum_{k=1}^{j-1} T_k$ worker j enters the game.

Figure 4

Game 2 Potential Outsiders; worker j enters the game.

t	ϑ	worker j (insider)		firm		
K_j	a			w_0 (j)		
	b	$A_0(j)$	$\longleftarrow$			
K_j + 1	a	$w_1(j)$				
	b		$\longrightarrow$	A_1 (j)		
.	.		.	.		
.	.		.	.		
.	.		.	.		
K_j + T-2	a			w_{T-2} (j)		
	b	$A_{T-2}(j)$	$\longleftarrow$			
K_j + T-1	a	$w_{T-1}(j)$				
	b		$\longrightarrow$	$A_{T-1}(j)$		
	c			$S_{T-1}(j)$	$\xrightarrow{Y}$	T_j=T;
K_j + T	a			$w_T(j)$	N $\downarrow$	restart with an
	b	$A_T(j)$	$\longleftarrow$			outsider (worker j+1)
K_j + T+1	a	$w_{T+1}(j)$				
	b		$\longrightarrow$	$A_{T+1}(j)$		
	c			$S_{T+1}(j)$	$\xrightarrow{Y}$	T_j=T+2;
K_j + T+1	a			$w_{T+2}(j)$	N $\downarrow$	restart
	b	$A_{T+2}(j)$	$\longleftarrow$			with an
.	.	.		.		outsider
.	.	.		.		(worker
.	.	.		.		j + 1)

The important thing to notice in figure 2 is that after a switch the original game reappears (with the new insider, worker j+1) and that the subgame beginning just before a possible switch (i.e. beginning at some stage ϑ = c) equals the subgame beginning just before the next possible switch.

To describe equilibrium in the game 2, the notation needs to be adjusted somewhat.

$T_j \in \{T, T+2, \ldots\}$	: number of rounds the firm plans to bargain with worker j.
$f_j := (A_0(j), w_1(j), \ldots, A_{Tj-2}, w_{Tj-1}),\ j = 1, 2, \ldots$	: worker j's strategy
$f := (f_1, f_2, \ldots)$	: workers' strategy
$g_j := (w_0(j), A_1(j), \ldots, w_{Tj-2}, A_{Tj-1}$ and $S_{Tj-1}),\ j = 1, 2, \ldots$	: firm's subgamestrategy during negotiations with worker j.
$g := (g_1, g_2, \ldots)$	: firm's strategy
$t \in \{0, 1, 2, \ldots\}$	: bargaining rounds
$\vartheta \in \{a, b, c\}$	: subdivision of a round t; at stage $\vartheta = a$ W_t is proposed, at the subsequent stage $\vartheta = b$ the reaction A_t is given and at the subsequent stage $\vartheta = c$ (where relevant) a switch decision S_t is taken.
$w_s(j) \in [0,1]$	: wage proposal in s-th round with worker j, $s = 0, 1, \ldots, T_j-1$
$A_s(j) \in \{Y,N\}$	: reaction on $W_s(j)$
$S_s(j) \in \{Y,N\}$	: switch decision in s-th round with worker j, $s = T-1, T+1, \ldots, T_j-1$.
$P(f_j,g_j) \in [0,1] \times \{K_j, K_j+1, \ldots, K_{j+1}-1\}$	: bargaining outcome for worker j when the firm plays g; and the worker f_j. If agreement is reached at $t \in \{K_j, K_j+1, \ldots, K_{j+1}-1\}$ then $P(f_j,g_j) = (w_t,t)$. If no agreement is reached (i.e. a switch occurs), then assign $P(f_j,g_j) = (0, K_{j+1}-1)$ to denote the worst that can happen to worker j.
$P(f,g) \in [0,1] \times \{0, 1, 2, \ldots\}$	: bargaining outcome for the firm when the firm plays g and the workers play f. Assign $P(f,g) = (w,\infty)$ for some $w \in [0,1]$ to denote perpetual disagreement.

$\succsim_{(j)}$: complete, reflexive and transitive preference relation j-th worker over set of ordered pairs $P(f_j, g_j)$.

$\succsim_2$: idem firm, over pairs $P(f,g)$.

Notice that the number of rounds that worker j may be in the game (T_j) can change as the workers' strategy f and the firm's strategy g changes.
By assumption $T_j \geq T$ and T_j may go to infinity to indicate that no switch occurs after worker j has entered the game.

3.2 Nash Equilibrium

As before the NE concept is not very appropriate to describe behaviour in the game 2: it cannot single out an outcome and it is subgame imperfect. The arguments to illustrate this can be found in section 2.2.

3.3 Perfect Equilibrium

The PE concept is as powerful in the bargaining game 2 as it was in the original game 1. Equilibrium wage proposals can be fully characterised for any stage the game can reach (for whatever reason); also it will be found that in equilibrium the firm will switch whenever an opportunity has arised.

Definition 3 (Perfect Equilibrium; Potential Outsiders)
A pair of strategies (f^*, g^*) is called a PE in the game 2 if:
A) if worker j has entered the game at $t = K_j$, then:

$$\forall t \in \{K_j,\ K_j+1,\ \dots\ ,\ K_{j+1}\},\ \forall \vartheta \in \{a,b\}$$

$$P(f_j^*|t_\vartheta,\ g_j^*|t_\vartheta) \succsim_{(j)} P(f_j|t_\vartheta,\ g_j^*|t_\vartheta),\ \forall f_j|t_\vartheta$$

B) $\forall t \in \{0,\ 1,\ 2\ ,\ \dots\ \},\ \forall \vartheta \in \{a,\ b,\ c\}$

$$P(f^*|t_\vartheta,\ g^*|t_\vartheta) \succsim_2 P(f^*|t_\vartheta,\ g|t_\vartheta),\ \forall g|t_\vartheta$$

Subgamestrategies are defined as in section 2.3.

The same assumptions on the players' preferences are made (time is valuable, wage is desirable for a worker and profit for the firm, continuity, stationarity and increasing compensation; see section 2). Furthermore it is assumed that all workers have equivalent preferences, denoted by $\underset{\sim}{>}_1$. Proposition 2 can now be extended to characterize PE in the game 2. Recall:

$$d_1(x) := \min_{(w,0) \underset{\sim}{>}_1 (x,1)} w$$

$$d_2(y) := \max_{(w,1) \underset{\sim}{>}_2 (y,2)} w$$

<u>Proposition 4</u> (PE in game 2)

A) (i) $y_s \in [0,1]$ is a PE wage proposal by the firm in the s-th bargaining round with the current insider, $s \in \{0, 2, \ldots, T_j-2\} \Leftrightarrow$

$y_0 = d_1(d_2(y_2))$
$y_2 = d_1(d_2(y_4))$
.
.
.
$y_{T-2} = d_1(d_2(y^*))$ with $y^* := \min \{y_0, y_{T-2}\}$
$y_s = y_{T-2}$ for $s \in \{T, T+2, \ldots, T_j-2\}$

(ii) $x_{s+1} \in [0,1]$ is a PE wage proposal by the current insider in the (s+1)-th bargaining round with the firm, $s \in \{0, 2, \ldots, T_{j-2}\}$ $\Leftrightarrow$

$x_1 = d_2(d_1(x_3))$
$x_3 = d_2(d_1(x_5))$
.
.
.
$x_{T-1} = d_2(d_1(x^*))$ with $x^* := d_1^{-1}(y^*)$
$x_{s+1} = x_{T-2}$ for $s \in \{T, T+2, \ldots, T_j-2\}$

(iii) $y_0 \leq y_{T-2}$

B) At least one sequence of wages $\{y_s\}$, $\{x_{s+1}\}$, $s = 0, 2, \ldots, T_j-2$ as described in A exists.

C) $s \in \{0, 2, \ldots T_j-2\}$; $j \in \{1, 2, 3, \ldots\}$:
(w,s) resp. (w,s+1) is a PE outcome of the subgame starting in the s-th resp. (s+1)-th bargain round with worker j $\Leftrightarrow$ $w = y_s$ resp. $w = x_{s+1}$.

Proof

As in the proof of proposition 2 the reappearance of games plays a crucial role: the original game reappears if a switch occurs and the subgame just before a possible switch reappears if a next possibility to switch occurs.

A) (i) and (ii)

($\Leftarrow$) Let y_s, x_{s+1}, $s \in \{0, 2, \ldots, T_j-2\}$ satisfy the right hand side conditions and consider the following pair of strategies: the firm proposes y_s in the s-th round, rejects any wage greater than x_{s+1} in the (s+1)-th round and switches if $y_0 \leq y_{T-2}$; the insider proposes x_{s+1} in the (s+1)-th round and rejects any wage smaller than y_s in the s-th round. From definition of d_1 and d_2 and the reappearance of games it follows that neither the firm nor the insider can improve upon the proposals and reactions in any possible stage of the game. Therefore the strategies form a PE.

($\Rightarrow$) To see that the conditions are necessary, the argument in the proof of proposition 2 (section 2) can be repeated. For the sake of brevity the details are omitted here.

A) (iii)

To see that $y_0 \leq y_{T-2}$, suppose $y_0 > y_{T-2}$. Then $y^* = y_{T-2}$ and substitution in part A(i) gives:

$$y_{T-2} = d_1(d_2(y_{T-2}))$$
$$y_{T-4} = d_1(d_2(y_{T-2})) = y_{T-2}$$
$$\vdots$$
$$y_0 = y_{T-2}$$ which contradicts $y_0 > y_{T-2}$

B) From A(i) and A(iii) $y^* = y_0$. Because the function d_1 and d_2 are continuous functions from [0,1] into [0,1], y^* are the fixed points of the compounded function $D_{12}(w) := d_1 \circ d_2 \circ d_1 \circ \dots \circ d_1 \circ d_2(w)$. From the existence of y^* the other wages y_s can be computed recursively. The same holds for x^* and x_{s+1}.

C) This part of the proposition follows from the construction of strategies in part A. □

As in section 2 the results are illustrated with the fixed discounting factor utility functions.

<u>Corollary 2</u> (fixed discounting factors)

Let all workers have utilities:

$$\begin{aligned} u_1(w,t) &= \delta_1^{\,t}\, w \text{ if } t \in \{K_j,\ K_j+1,\ \dots,\ K_{j+1}-1\} \\ &= 0 \qquad \text{elsewhere, i.e. if he does not enter the game } (t < K_j) \\ &\qquad\quad \text{or if a switch occurs } (t \geq K_{j+1}) \end{aligned}$$

and let the firm's utilityfunction be u_2:

$$u_2(w,t) = \delta_2^{\,t}(1-w) \qquad , \ \delta_1\delta_2 \neq 1.$$

Then the bargaining game 2 has unique PE wages:

$$y_s = \frac{\delta_1(1-\delta_2)\ (1-(\delta_1\delta_2)^{(T-s-2)/2})}{(1 - \delta_1\delta_2)} + \delta_1^{\,(T-s)/2}.\ \delta_2^{\,(T-s-2)/2}.y_0$$

Particularly:

$$y_0 = \frac{\delta_1(1-\delta_2)\ (1 - \delta_1^{\,k}\ \delta_2^{\,k})}{(1-\delta_1\delta_2)\ (1 - \delta_1^{\,k+1}\ \delta_2^{\,k})} \qquad , \ k := \frac{T-2}{2}$$

y_s is the unique PE wage proposals by the firm ($s = 0, 2, \dots, T-2$).

$$x_s = \frac{1}{\delta_1}\, y_{s-1} \text{ or equivalently } x_s = 1 - \delta_2 + \delta_2\, y_{s+1}$$

x_{s+1} is the unique PE wage proposals by the insider ($s = 1, 3, \dots, T-1$)

Proof

The function d_1 and d_2 are: $d_1(x_s) = \delta_1 x_s$

$$d_2(y_s) = 1 - \delta_2 + \delta_2 y_s$$

Direct application of proposition 4 leads to:

$$y_s = \delta_1[1-\delta_2 + \delta_1\delta_2 - \delta_1\delta_2^2 + \delta_1^2 \delta_2^2 - \dots$$
$$- \delta_1^{(T-s-4)/2} \delta_2^{(T-s-2)/2} + (\delta_1\delta_2)^{(T-s-2)/2} . y_0]$$

It is easily verified that this equation equals the equation for y_s above. The formula for y_0 follows with $s = 0$. Because proposition 4 implies $x_s = d_2(y_{s+1})$ and $y_s = d_1(x_{s+1})$, the results for x_s are evident.

□

Several remarks must be made at this point.

1. For equal discountfactors $\delta_1 = \delta_2 = \delta < 1$, the unique PE outcome is $(y_0, 0)$, with $y_0 = \frac{\delta}{1 + \delta} \left[\frac{1-\delta^{T-2}}{1-\delta^{T-1}}\right]$. This is the main result in Shaked and Sutton (1984)[9].
   Corollary 2 extends this, dealing with the case of different discount factors $\delta_1 \neq \delta_2$ and describing explicitly equilibrium behaviour as long as no agreement is reach for whatever (irrational) reasons. Moreover, proposition 4 applies to a braoder class of preferences (see note 7).

2. If T goes to infinity (i.e no switch is ever allowed for), the insider gets monopoly power and indeed the Bilateral Monopoly result of Rubinstein (1982) emerges: $y = \frac{\delta_1(1-\delta_2)}{1 - \delta_1 \delta_2}$; $x = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}$ (see corollary 1).
   If on the other hand T = 2 (i.e. the firm is allowed to switch whenever it is his turn react), the firm can get full profit and PE wages are zero:

$$y = 0 \ ; \ x = 0$$

Notice that $w = 0$ is precisely the worker's reservation wage; this competitive equilibrium is here not the result of instantaneous competition between workers (as in a Walras labor market where a firm can propose wages to several workers at the same time) but rather of sequential competition between workers (the firm threatens to switch to an outsider, leaving the insider with unemployment and utility zero). Notice further that the equilibrium wages rise in T: the longer the firm has to wait on a possible switch, the better is the bargaining position of the insider.

3. If $\delta_1\delta_2$ approaches unity (i.e. nor the firm nor the workers are impatient) the equilibrium wage $y_0 = \frac{T-1}{2T}$. Again for $T = 1$ the reservation wage $w = 0$ arises. For $T = \infty$, $y_0 = \frac{1}{2}$ is precisely the axiomatic Nash solution of the corresponding static problem[10].

4. If $\delta_1 = 1$ and $\delta_2 < 1$ (i.e. only the firm is impatient) then $y_s = x_s = 1$, $\forall s$, thus the worker can attain full wage in equilibrium. Also, if $\delta_1 < 1$ and $\delta_2 = 1$ the firm gets full profit: $y_s = x_s = 0$.

Another result that can be derived from proposition 4 extends the model to the case in which the reaction time of the firm respectively the workers is different (due to for example having to discuss proposals with third parties).

Corollary 3

Let Δ_1 be the reaction time of the workers (i.e. the length of the first, third, fifth. etc. ... bargaining round) and Δ_2 of the firm.
The game 2, modified in this way and with preferences as in corollary 2, has unique PE wage proposals. They can be computed by replacing the discount factors δ_i by $\delta_i^{\Delta_i}$.

Proof

The functions d_1 and d_2 are now defined as:

$$d_1(x) := \min_{(w,0) \succsim_1 (x,\Delta_1)} w = \delta_1^{\Delta_1} x$$

$$d_2(y) := \max_{(w,\Delta_1) \succsim_1 (y,\ \Delta_1 + \Delta_2)} w = 1 - \delta_2^{\Delta_2} + \delta_2^{\Delta_2} y$$

Applying proposition 4 (corrolary 2 with $\delta_i^{\Delta_i}$ in stead of δ_i) leads directly to the result

□

PE wages are more favourable to the insider the higher his $\delta_1^{\Delta_1}$ or the lower the $\delta_2^{\Delta_2}$ of the firm; thus a worker who is less impatient (i.e. δ_1 larger) and who can react quicker on a proposal has a bargaining advantage over a worker who is more impatient and reacts shower. The same holds (mutatis mutandis) for the firm.[11]

4 Bilateral Monopoly, Incomplete Information

4.1 Incomplete Information in the Basic Model

In this section the assumption of complete information, which was essential in the previous sections, is dropped.
Consider once again bargaining game 1 as described in section 2, figure 1. Following Rubinstein (1985b) it is now assumed the firm (player 2 in the game) is of two possible types. Either it is weak or it is strong (in the sense of relatively impatient resp. relatively patient). The worker (player 1 in the game) does not know the firm's actual type, but nevertheless he has some belief, say p(t), at round t that the firm is weak. One might say that 1-p(t) is the reputation of the firm of being strong.

Before describing equilibrium in this incomplete information game 1 the notation of section 2 needs to be extended somewhat. Lower bars are used for the weak firm and upper bars for the strong firm.

$f := (A_0, w_1, A_2, w_3, \dots)$: worker's strategy

$$\left.\begin{aligned} g &= \underline{g} := (\underline{w}_0\ \underline{A}_1\ \underline{w}_2\ \underline{A}_3, \dots) \text{ if weak} \\ &= \bar{g} := (\bar{w}_0, \bar{A}_1, \bar{w}_2, \bar{A}_3, \dots) \text{ if strong} \end{aligned}\right\}$$: firm's strategy

$f|t_\vartheta$: worker's subgamestrategy at stage ϑ of round t

$$\left.\begin{aligned} g|t_\vartheta &= \underline{g}|t_\vartheta \text{ if weak} \\ &= \bar{g}|t_\vartheta \text{ if strong} \end{aligned}\right\}$$: firm's subgamestrategy at stage ϑ of round t

$$\left.\begin{aligned} &P(f, \underline{g}, \bar{g}) := \langle P(f,\underline{g}), P(f,\bar{g}) \rangle \\ &\text{with } P(f,g) \in [0,1] \times \{0, 1, 2, \dots\} \end{aligned}\right\}$$: bargaining outcome when the worker plays f and the firm plays $\underline{g}$ if its is weak and $\bar{g}$ if it is strong

$p.P(f,\underline{g}) \oplus (1-p).P(f,\bar{g})$: lottery between the outcome $P(f,\underline{g})$ with probability p and $P(f,\bar{g})$ with probability 1-p, $p \in [0,1]$.

$\succsim_1$: preference relation of the worker over lotteries between two outcomes $P(f,\underline{g})$ and $P(f,\bar{g})$, both $\in [0,1] \times \{0, 1, 2, \dots\}$

$\succsim_2$: preference relation of the firm over outcomes $P(f,g) \in [0,1] \times \{0, 1, 2, \dots\}$: weak firm: $\succsim_{\underline{2}}$, $\underline{g}$; strong firm: $\succsim_{\bar{2}}$, $\bar{g}$.

Definition 4 (the firm's type)

Let, as before, $d_2(y) := \max_{(w,0) \succsim_2 (y,1)} w$, $2 \in \{\underline{2},\bar{2}\}$, $\underline{2} \neq \bar{2}$.

If $\forall y \in [0,1]$ such that $d_{\underline{2}}(y) < 1$: $d_{\bar{2}}(y) < d_{\underline{2}}(y)$, then the type $\underline{2}$ is said to be weak and the type $\bar{2}$ is said to be strong.

The definition says that if the firm can obtain a wage y in the next round, it shall require a smaller wage (i.e. a higher profit) today when it's type is strong than it would require when it's type were weak.
If for example $\succsim_2$ can be represented with $u_2(w,t) = \delta_2{}^t(1-w)$ with $\delta_2 = \alpha \in [0,1]$ for the weak firm and $\delta_2 = \beta \in [0,1]$ for the strong firm, then $d_2(y) = 1 - \delta_2 + \delta_2\, y$ and weakness and strength is defined as: $\alpha < \beta$; the discounting factor of the weak firm is smaller (i.e. it's discount-rate is bigger, see note 5).

4.2 Nash Equilibrium

The definition of NE is straightforward.

Definition 5 (Nash equilibrium under incomplete information) $(f^*, g^*, \bar{g}^*)$ is called a Nash equilibrium (NE) if

A) $p_0\, P(f^*, g^*) \oplus (1-p_0).P(f^*, \bar{g}^*) \succsim_1 p_0.P(f, g^*) \oplus (1-p_0).P(f, \bar{g}^*), \quad \forall f$

B) (i) $P(f^*, \bar{g}^*) \succsim_2 P(f^*, g) \quad , \forall g$

(ii) $P(f^*, \bar{g}^*) \succsim_{\bar{2}} P(f^*, \bar{g}) \quad , \forall \bar{g}$

Notice that for $p_0 = 1$ part A and B(i) define NE in the complete information game 1 between the worker and a weak firm; also for $p_0 = 0$ part A and B(ii) define NE between the worker and a strong firm (see definition 1).

As in the complete information case (see proposition 1), the set of NE outcomes can be quite large.

Proposition 5 (weakness NE)

$(f^*, g^*, \bar{g}^*)$ NE such that $P(f^*, g^*, \bar{g}^*) = \langle (\underline{w}, \underline{t}), (\bar{w}, \bar{t}) \rangle$

$\Leftrightarrow$ A) $\quad p_0.(\underline{w}, \underline{t}) \oplus (1-p_0).(\bar{w}, \bar{t}) \succsim_1 (0,0)$

B) (i) $(\underline{w}, \underline{t}) \succsim_2 (1,0) \wedge (\underline{w}, \underline{t}) \succsim_2 (\bar{w}, \bar{t})$

(ii) $(\bar{w}, \bar{t}) \succsim_{\bar{2}} (1,0) \wedge (\bar{w}, \bar{t}) \succsim_{\bar{2}} (\underline{w}, \underline{t})$

Proof

($\Longrightarrow$) If one of the conditions is not satisfied, $(f^*, g^*, \bar{g}^*)$ cannot be a NE: if A is not true the worker can improve by accepting anything at $t = 0$; if B(i) is not true, the weak firm can improve by proposing $w = 1$ at $t = 0$ or by playing the strong type; if B(ii) is not true, the strong firm can improve by proposing $w = 1$ at $t = 0$ or by playing the weak type.

($\Longleftarrow$) Suppose the conditions hold and let $\underline{t} \le \bar{t}$ (changing the role of $\underline{t}$ and $\bar{t}$ gives the result for $\underline{t} \ge \bar{t}$). A NE $(f^*, g^*, \bar{g}^*)$ leading to $\langle (\underline{w}, \underline{t}), (\bar{w}, \bar{t}) \rangle$ is constructed as follows (cf. proof proposition 1): Until $\underline{t}$ is reached the worker constantly proposes full wage and rejects anything less. In rounds $\underline{t}$ to $\bar{t}-1$ he proposes $\underline{w}$ and rejects anything less. From $\bar{t}$ onward he proposes $\bar{w}$ and rejects anything less.

The weak firm proposes full profit until $\underline{t}$ is reached and rejects any lower profit. From $\underline{t}$ onwards it proposes $\underline{w}$ and reject any higher wage. In the same way, the strong firm proposes full profit until $\bar{t}$ and $\bar{w}$ from $\bar{t}$ onwards.

It is easily checked that none of the players can improve ex ante on his (expected) outcome. □

For $\underline{t} = \bar{t} = 0$ the conditions A and B are valid for every $\underline{w} = \bar{w} = w$ with $w \in [0,1]$; thus every wage in the first bargaining round can be interpreted as a Nash wage and NE is not a powerful equilibrium concept. Also, as before, NE may be time inconsistent (see example 1).

Therefore we need the concept of subgame-perfect equilibrium for incomplete information games. Since Kreps and Wilson (1982a) this concept is usually called "sequential equilibrium".

4.3 Sequential Equilibrium

The worker's initial belief (i.e. the firm's initial reputation of being weak or strong) and the way in which this belief can change during negotiations, play an important role in describing subgame perfect equilibrium in the incomplete information game under consideration.

Consider the worker's belief that the firm is weak after the firm's proposal w_t, $t = 0, 2, 4, \ldots$. The current belief is some function of the previous belief and of the moves the firm has made in the last two rounds:

$$p(t) = h(p(t-2), A_{t-1}, w_t) \qquad , \; t = 2, 4, 6, \ldots$$

$$p(0) = h(p_0, w_0)$$

with p_0: worker's belief before the game starts.

Generally only socalled plausible beliefs are considered (see Harsanyi (1967/1968), Kreps and Wilson (1982a) and Rubinstein (1985b)).

Definition 6 (plausible beliefs)

The worker's beliefs $\{p(t)\}_{t=0,2,4,\ \dots}$ are said to be plausible if:

A) the worker doesn't change his mind, if he cannot distinguish between the weak firm's moves $(\underline{A}_{t-1}, \underline{w}_t)$ and the strong firm's moves $(\bar{A}_{t-1}, \bar{w}_t)$: $\underline{A}_{t-1} = \bar{A}_{t-1} = N \wedge \underline{w}_t = \bar{w}_t = w_t \Longrightarrow p(t) = p(t-2)$

B) the worker concludes that the firm is of certain type, if the observed moves are only compatible with that type: $0 < p(t-2) < 1$:

(i) $\underline{A}_{t-1} = N \wedge \underline{w}_t = w_t \wedge (\bar{A}_{t-1} = Y \vee \bar{w}_t \neq w_t) \Longrightarrow p(t) = 1$

(ii) $\bar{A}_{t-1} = N \wedge \bar{w}_t = w_t \wedge (\underline{A}_{t-1} = Y \vee \underline{w}_t \neq w_t) \Longrightarrow p(t) = 0$

C) the worker's conclusion that the firm is of a certain type is definite:

(i) $p(t-2) = 1 \Longrightarrow p(t) = 1$

(ii) $p(t-2) = 0 \Longrightarrow p(t) = 0$

Part A and B of this definition follow from Bayes' rule[12]; part c also seems a quite reasonable restriction on the sequence of beliefs.

To define sequential equilibrium (SE), the worker's belief at stage ϑ of round t is represented by $p(t_\vartheta)$ (see figure 1, section 2 for the players' moves at stage ϑ of round t).

Figure 5

Evolution of reputation

t	ϑ	$p(t_\vartheta)$
0	a	p_0
	b	$p(0)$
1	a	$p(0)$
	b	$p(0)$
2	a	$p(0)$
	b	$p(2)$
3	a	$p(2)$
	b	$p(2)$
4	a	$p(2)$
	b	$p(4)$
.	.	.
.	.	.

Definition 7 (Sequential equilibrium; PE under incomplete information)
A triple $(f^*, g^*, \bar{g}^*)$ and a sequence of beliefs $\{p(t)\}_{t = 0, 2, \ldots}$ are called a sequential equilibrium (SE) if the beliefs are plausible and if $\forall t \in \{0, 1, 2, \ldots\}$, $\forall \vartheta \in \{a,b\}$

A) $p(t_\vartheta).P(f^*|t_\vartheta, g^*|t_\vartheta) \oplus (1-p(t_\vartheta)).P(f^*|t_\vartheta, \bar{g}^*) \succsim_1$

$p(t_\vartheta).P(f|t_\vartheta, g^*|t_\vartheta) \oplus (1-p(t_\vartheta)).P(f|t_\vartheta, \bar{g}^*|t_\vartheta)$, $\forall f|t_\vartheta$

B) (i) $P(f^*|t_\vartheta, g^*|t_\vartheta) \succsim_{\underline{2}} P(f^*|t_\vartheta, g|t_\vartheta)$, $\forall g|t_\vartheta$

(ii) $P(f^*|t_\vartheta, \bar{g}^*|t_\vartheta) \succsim_{\bar{2}} P(f^*|t_\vartheta, \bar{g}|t_\vartheta)$, $\forall \bar{g}|t_\vartheta$

The next proposition compares PE wage proposals (in the complete information game) with SE wage proposals (in the incomplete information game). The worker is better off with a PE wage if the firm is weak and with a SE wage if the firm is strong. From the point of view of the firm this means that if it is weak it may take advantage of it's reputation of being strong and if it is strong it may suffer from it's reputation of being weak.

<u>Proposition 6</u> (incomplete information is advantageous for the weak firm and disadvantageous for the strong firm).

Let $y(2)$ and $x(2)$ be PE wage proposals by the firm and the worker in the complete information game with the firm being weak ($\underline{2}$) respectively strong ($\bar{2}$). (see section 2, prop. 2). Any SE outcome $\langle (\underline{w},\underline{t}), (\bar{w},\bar{t}) \rangle :=$ $\langle P(f^*,\underline{g}^*), P(f^*,\bar{g}^*) \rangle$ in the incomplete information game satisfies:

$$(y(\underline{2}),0) \succsim_1 (\underline{w},\underline{t}) \quad \text{and} \quad (\underline{w},\underline{t}) \succsim_{\underline{2}} (y(\underline{2}),0);$$

$$(\bar{w},\bar{t}) \succsim_1 (y(\bar{2}),0) \quad \text{and} \quad (y(\bar{2}),0) \succsim_{\bar{2}} (\bar{w},\bar{t}).$$

Similarly for a SE outcome $\langle (\underline{v},\underline{s}), (\bar{v},\bar{s}) \rangle$ in the subgame starting at $t = 1$:

$$(x(\underline{2}),1) \succsim_1 (\underline{v},\underline{s}) \quad \text{and} \quad (\underline{v},\underline{s}) \succsim_{\underline{2}} (x(\underline{2}),1);$$

$$(\bar{v},\bar{s}) \succsim_1 (x(\bar{2}),1) \quad \text{and} \quad (x(\bar{2}),1) \succsim_{\bar{2}} (\bar{v},\bar{s}).$$

<u>Proof</u>

Since it is not rational for the worker to accept wages smaller than $y(\bar{2})$ and not rational for the firm to propose wages greater than $y(\underline{2})$, any SE wage y_t suggested by the firm and accepted by the worker must satisfy: $y(\bar{2}) \le y_t \le y(\underline{2})$, $t = 0, 2, 4, \ldots$. Also for a SE wage x_t suggested by the worker and accepted by the firm: $x(\bar{2}) \le x_t \le x(\underline{2})$, $x = 1, 3, 5, \ldots$ From proposition 2 it follows that

$$y(2) = d_1(x(2)) := \min_{(w,0) \succsim_1 (x(2),1)} w \quad \text{and}$$

$$x(2) = d_2(y(2)) := \max_{(w,1) \succsim_2 (y(2),2)} w \quad , \; 2 \in \{\underline{2},\bar{2}\}.$$

From this it can be easily verified that $\langle (\underline{w},\underline{t}), (\bar{w},\bar{t}) \rangle$ cannot be a SE outcome if one of the conditions does not hold; if for example $(y(\underline{2}),0) <_1$

$(\underline{w},\underline{t})$ or $(\underline{w},\underline{t}) <_{\underline{2}} (y(\underline{2}),0)$ then $\underline{w} > y(\underline{2})$, thus violating $y_t \le y(\underline{2})$. Also if $(\bar{w},\bar{t}) <_1 (y(\bar{2}),0)$ or $(y(\bar{2}),0) <_{\bar{2}} (\bar{w},\bar{t})$ then $\bar{w} < y(\bar{2})$, thus violating $y_t \ge y(\bar{2})$

□

To construct SE strategies and to characterize SE outcomes the following functions are essential (cf. d_1 and d_2 in section 2):

$$d_{1p}(x,z) := \min_{(w,0) \succsim_1 p(x,1) \oplus (1-p)(z,2)} w \qquad , x, z \in [0,1];\ p \in [0,1]$$

$$d_2(y) := \max_{(w,1) \succsim_2 (y,2)} w \qquad , y \in [0,1];\ 2 \in \{\underline{2},\bar{2}\}$$

$d_{1p}(x,z)$ gives the minimum wage the worker will accept now given some agreement x in the next round if the firm is weak and some agreement z two rounds later if the firm is strong. $1 - d_2(y)$ gives the minimum profit the firm will accept now given some agreement y in the next round.

Although the SE concept excludes incredible threats and is thereby time consistent on as well as off the equilibrium path, it can not in general single out a typical outcome of the bargaining game (as the PE concept could in the complete information case, see section 2 proposition 2, corollary 1).
The set of SE outcomes depends heavily on further assumptions on the plausible beliefs; plausibility allows a free choice of the worker's belief after an unexpected move by the firm. For example the worker may have the prejudgement that the firm is weak whenever it deviates from a certain (equilibrium) path $(g,\bar{g})$. In that case many SE outcomes are possible:

<u>Proposition 7</u> (SE under optimistic beliefs)

A) $\forall y \in [0,1]$ such that $(y,0) \precsim_1 (x(\underline{2}),1) \wedge (y,0) \succsim_1 (x(\bar{2}),1)$
$\exists$SE such that either

$$P(f^*, g^*, \bar{g}^*) = \langle (y,0), (y,0) \rangle \text{ or}$$

$$P(f^*, g^*, \bar{g}^*) = \langle (d_{\underline{2}}(y), 1), (y, 2) \rangle$$

B) Similarly in the subgame starting at t = 1:
$\forall x \in [0,1]$ such that $(x,1) \precsim_2 (y(\bar{2}),2) \wedge (x,1) \succsim_{\bar{2}} (y(\underline{2}),2)$
$\exists$SE such that either

$$P(f^*, g^*, \bar{g}^*) = \langle (x,1), (x,1) \rangle \text{ or}$$

$$P(f^*, g^*, \bar{g}^*) = \langle (x,1), (d_{\underline{2}}^{-1}(x),2) \rangle$$

Proof

A) Suppose for some $y \in [0,1]$, $(x(\bar{2}),1) \precsim_1 (y,0) \precsim_1 (x(\underline{2}),1)$ and let $p_0 \in [0,1]$ denote the worker's initial belief (before the firm's first proposal). If $p_0 = 0$ the worker (thinks he) knows the firm is strong. Because this conclusion is definite the game then equals the complete information game and $y = y(\bar{2})$ is the unique PE proposal supported by the PE strategies defined in the proof of proposition 2 (the strategies $f^* = f^*$, $g^* = g^*$ and $\bar{g}^* = g^*$ are a SE leading to $\langle (y(\bar{2}),0), (y(\bar{2}),0) \rangle$). Similarly $(y(\underline{2}),0)$ is the unique equilibrium outcome if $p_0 = 1$ and is supported by the equilibrium strategies of proposition 2.
If $p_0 \in (0,1)$, then consider the following beliefs and strategies:
Beliefs: plausible and in addition optimistic; i.e.

$$p(t) = 1 \quad \text{if } A_{t-1} \notin \{\underline{A}_{t-1}, \bar{A}_{t-1}\}$$
$$\text{or } w_t \notin \{\underline{w}_{t-1}, \bar{w}_{t-1}\}$$
$$, t = 0, 2, 4, \ldots$$

Strategies: $f^* = (A_0, w_1, A_2, w_3, \ldots)$

$$g^* = (\underline{w}_0, \underline{A}_1, \underline{w}_2, \underline{A}_3, \ldots)$$

$$\bar{g}^* = (\bar{w}_0, \bar{A}_1, \bar{w}_2, \bar{A}_3, \ldots)$$

with for t = 0, 2, 4, ... as long as $p(t) = p_0$:

$$f^* : A_t = Y \text{ if } w_t \geq d_{1p(t)}\,(d_{\underline{2}}(y),\ y)$$

$$= N \text{ if } w_t < d_{1p(t)}\,(d_{\underline{2}}(y), y)$$

$$w_{t+1} = d_{\underline{2}}(y)$$

$$\underline{g}^* : \underline{w}_t = y$$

$$\underline{A}_{t+1} = Y \text{ if } w_{t+1} \leq d_{\underline{2}}(y)$$

$$= N \text{ if } w_{t+1} > d_{\underline{2}}(y)$$

$$\bar{g}^* : \bar{w}_t = y$$

$$\bar{A}_{t+1} = Y \text{ if } w_{t+1} \leq d_{\bar{2}}(y)$$

$$= N \text{ if } w_{t+1} > d_{\bar{2}}(y)$$

and as soon as $p(t) = 0$ or $p(t) = 1$ the corresponding complete information PE strategies are played, see section 2.3.

Obviously, if $y \geq d_{1p_0}(d_{\underline{2}}(y), y)$ then the strategies above lead to the outcome $\langle (y,0), (y,0) \rangle$ and if $y < d_{1p_0}(d_{\underline{2}}(y), y)$ then $\langle (d_{\underline{2}}(y),1), (y,2) \rangle$ is the outcome, because $d_{\underline{2}}(y) > d_{\bar{2}}(y)$ by definition of the firm's type.
To see that the beliefs and strategies are a SE, first notice that from the moment $p(t)$ becomes 0 or 1 onwards no deviations are profitable (SE moves coincide with complete information PE moves). As long ad $p(t) = p_0$, the worker's optimism prevents the firm to deviate: any deviation leads to $p(t) = 1$ and thus to the weak outcome which is worse for both types.
Finally the worker never has an incentive to deviate either; it is easily verified that the expected outcome after a deviation is no better than the equilibrium subgame outcome.

B) In a similar way a SE with optimistic beliefs can be constructed for the subgame starting at $t = 1$ for any x satisfying the conditions given. This part of the proposition equals the proposition 3 in Rubinstein (1985b).

□

Optimistic beliefs as defined in the proof above are assumed by several authors (e.g. Fudenberg and Tirol (1983) and Perry (1986)). They serve as a kind of threat to the firm: if it does not stick on the equilibrium path, the SE outcome for the remainder of the game is the complete information PE outcome of the game between the worker and the weak firm. This is not an attractive prospect for the firm.

Although optimistic beliefs deter best, there are good reasons why they are not appropriate to describe reality. Why should for example the worker not conclude the firm is strong if it has revealed its preferences? Or what arguments does he have to conclude weakness if the firm insists on a higher profit?

It seems more appropriate to assume the worker adjusts his belief in the direction that rationalizes the firm's (unexpected) behaviour:

Definition 8 (rationalizing beliefs)

The worker's beliefs $\{p(t)\}_{t=0,2,4,\ ...}$ are said to be rationalizing if:

A) the worker concludes that the firm is strong if it rejects a proposal and it's counterproposal is worse for the weak firm but at least as good for the strong firm:

$p(t-2) \neq 1 \wedge A_{t-1} = N \wedge (w_t, t) \underset{\bar{2}}{<} (w_{t-1}, t-1) \wedge (w_t, t) \underset{\sim\bar{2}}{>} (w_{t-1}, t-1)$

$\Rightarrow p(t) = 0.$

B) the firm's insistence cannot be an indication that it is more likely to be weak:

$A_{t-1} = N \wedge (w_t, t) \underset{\sim\underline{2}}{>} (w_{t-1}, t-1) \wedge (w_t, t) \underset{\sim\bar{2}}{>} (w_{t-1}, t-1)$

$\Rightarrow p(t) \leq p(t-2)$

In Rubinstein (1985b) it is shown that under rationalizing beliefs SE wages can be characterised as PE wages were characterised in section 2 (proposition 2).

Proposition 8 (SE under rationalizing beliefs).

A) Let beliefs be rationalizing and assume the worker accepts a proposal w_t if he is indifferent between w_t and the expected outcome after having rejected w_t and also assume the firm is not allowed to propose anything less than $y(\bar{2})$ (i.e. the wage it would propose when strong).

Finally, let $p \in (0,1)$ denote the worker's current belief that the firm is weak (see figure 5) and let x and $y \in [0,1]$ satisfy $x = d_{\underline{2}}(y) \wedge y = d_{1p}(x,y)$.

Then for $t \in \{0, 2, 4, \dots\}$:

(i) $y > y(\bar{2}) \Rightarrow \langle (y,t), (y,t) \rangle$ and $\langle (x,t+1), (y,t+2) \rangle$ are the only SE outcomes of the (sub-) game starting at t

(ii) $y < y(\bar{2}) \Rightarrow \langle (y(\bar{2}),t), (y(\bar{2}),t) \rangle$ is unique SE outcome of the game starting at t

Similarly for $t + 1 \in \{1, 3, 5, \dots\}$:

(i) $y > y(\bar{2}) \Rightarrow \langle (x,t+1), (y,t+2) \rangle$ is unique SE outcome of the game starting at t+1.

(ii) $y < y(\bar{2}) \Rightarrow \langle (x(\bar{2}),t+1), (x(\bar{2}),t+1) \rangle$ is unique SE outcome of the game starting at t+1.

B) $\forall p \in (0,1)$ at least one pair $(x,y) \in [0,1] \times [0,1]$ satisfies $y = d_{1p}(x,y) \wedge x = d_{\underline{2}}(y)$.

The proof of proposition 8 is rather extensive and can be found in Rubinstein (1985b).

The example with fixed discounting factors illustrates how the propositions 7 and 8 work out. The relation between the firm's initial reputation and the corresponding equilibria is made explicit.

Example, continued

Consider once again the utility functions:

$$u_1(w,t) = \delta_1{}^t w \quad : \text{discounted wage}$$

$$u_2(w,t) = \delta_2{}^t (1-w) : \text{discounted profit.}$$

Let $\delta_1 = \delta$ and $\delta_2 = \alpha$ for the weak firm and $\delta_2 = \beta$ for the strong firm $(0 \le \delta < 1 \wedge 0 \le \alpha < \beta \le 1)$.
Here:

$$d_{1p}(x,z) = p\,\delta\,x + (1-p)\,\delta^2 z \qquad , p \in (0,1)$$

$$d_{\underline{2}}(y) = 1 - \alpha + \alpha y$$

$$x(\underline{2}) = \frac{1-\alpha}{1-\alpha\delta} \qquad , x(\bar{2}) = \frac{1-\beta}{1-\beta\delta} \quad ,$$

$$y(\underline{2}) = \frac{\delta(1-\alpha)}{1-\alpha\delta} \qquad , y(\bar{2}) = \frac{\delta(1-\beta)}{1-\beta\delta}$$

$$y \gtreqless d_{1p}(d_{\underline{2}}(y),y) \Leftrightarrow y \gtreqless \frac{p\delta(1-\alpha)}{1-p\alpha\delta-(1-p)\delta^2} \qquad =: y^*$$

$$\Leftrightarrow p \lesseqgtr \frac{y^*}{\delta\left[1-\alpha+\alpha y^* + (1-p)\delta y^*\right]} =: p^*$$

Proposition 7 implies that $\forall y \in \left[\frac{\delta(1-\beta)}{1-\beta\delta}, \frac{\delta(1-\alpha)}{1-\alpha\delta}\right]$ there is a SE with optimistic beliefs such that $\langle (y,0), (y,0) \rangle$ is the outcome if the firm's initial reputation of being strong is large (i.e. $y \ge y^*$, i.e. $p \le p^*$ and $\langle (d_{\underline{2}}(y),1), (y,2) \rangle$ is the outcome if this reputation is lower (i.e. $y < y^*$, i.e. $p > p^*$).
Part B of proposition 7 implies that
$\forall x \in \left[1 - \alpha + \alpha \frac{\delta(1-\beta)}{1-\beta\delta}, 1 - \beta + \beta \frac{\delta(1-\alpha)}{1-\alpha\delta}\right]$ there is a SE with optimistic beliefs such that $\langle (x,1), (x,1) \rangle$ or $\langle (x,1), (d_{\underline{2}}^{-1}(x),2) \rangle$ is the outcome with

$$d_{\underline{2}}^{-1}(x) = 1 - \frac{1-x}{\alpha} \text{ if } \alpha \ne 0$$

$$= 1 \qquad \text{if } \alpha = 0$$

It is interesting to note that the interval $\left[\frac{\delta(1-\beta)}{1-\beta\delta}, \frac{\delta(1-\alpha)}{1-\alpha\delta}\right]$ is never empty, but the interval $\left[1 - \alpha + \alpha \frac{\delta(1-\beta)}{1-\beta\delta}, 1 - \beta + \beta \frac{\delta(1-\alpha)}{1-\alpha\delta}\right]$ may be empty:[13]

$$\forall\alpha, \beta, \delta \in [0,1] :$$

$$\left[0 \leq \alpha < \beta < 1 \Rightarrow \frac{\delta(1-\beta)}{1-\beta\delta} < \frac{\delta(1-\alpha)}{1-\alpha\delta}\right]$$

Thus if $(\alpha + \beta)\delta < 1$, then:

$$1 - \alpha + \alpha \frac{\delta(1-\beta)}{1-\beta\delta} > 1 - \beta + \beta \frac{\delta(1-\alpha)}{1-\alpha\delta}$$

Thus Rubinstein's assertion that proposition 7, part B "demonstrates that the set of SE outcomes is very large" (Rubinstein (1985), p. 1159) is only valid for values of α, β and δ with $(\alpha + \beta)\ \delta \geq 1$.

5 Policy Bargaining in a Dynamic Economy

5.1 The Bilateral Monopoly Policy Model

Consider the system's representation of an economy:

$$x(t) = F(t, x(t-1), u(t-1))$$

$$x(0) = x_0$$

with $t \in T := \{1, 2, \ldots, t_f\}$: planperiod
$x(0) \in X^0$:= set of initial states
$x(t) \in X^t$:= set of possible states in period $t \in T$
$u(t-1) \in U^{t-1}$:= set of feasible controles in period $t-1$, $t \in T$
$F : T \times X^{t-1} \times U^{t-1} \longrightarrow X^t$

There are two decisionmakers in this economy. Each of them has the control over the set of variables $u_i(t-1)$ $(i = 1, 2, t \in T)$, subset of
$u(t-1) := (u_1(t-1), u_2(t-1)) \in U^{t-1} := U_1^{t-1} \times U_2^{t-1}$.

Controls depend on the information available:

$$u_i(s) = G_i(s; x(0); u(0), \ldots, u(s-1))$$

$$u_i(0) = u_{i0}$$

with $s \in S := \{1, 2, \ldots, t_f-1\}$

$$G_i : S \times X^0 \times \prod_{k=0}^{s-1} U^k \longrightarrow U_i^s$$

$i \in I := \{1,2\}$

The government is decisionmaker 1 (i=1) and the private sector decisionmaker 2 (i=2).

The decisionmakers are the players in a non-cooperative bargaining game in which they try to come to an agreement on the controldecisions to be made[14]. The bargaining process resembles the Rubinsteinmodel of section 2 (see Stefanski and Cichocki (1986)):

Figure 6

Game 3 Policy Bargaining, finite planperiod (t_f even)

t	ϑ	government		private sector
0	a			$\tilde{u}^0$
	b	A_0	$\longleftarrow$	
1	a	$\tilde{u}^1$		
	b		$\longrightarrow$	A_1
2	a			$\tilde{u}^2$
	b	A_2	$\longleftarrow$	
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
t_f-1	a	$\tilde{u}^{t_f-1}$		
	b		$\longrightarrow$	A_{t_f-1}
t_f		total disagreement: $\tilde{u}^{t_f}$		

In the first bargaining round the private sector makes a proposal concerning all present and future control variables $\tilde{u}^0 = (u(0), \ldots , u(t_f-1))$ with $u(s) \in U^s$, $s = 0, 1, 2, \ldots , t_f-1$. If the government accepts $\tilde{u}^0$ by playing $A_0 = Y$, the game ends and the agreed controls are executed. If however the government rejects $\tilde{u}^0 (A_0 = N)$, then disagreement controls are executed in the first round: $u_i(0) = u_i^d(0)$, $i = 1, 2$ and the bargaining contiunues with a proposal $\tilde{u}^1 := (u(0), u(1), \ldots , u(t_f-1))$ by the government with $u(0) = u^d(0)$; $u(s) \in U^s$, $s = 1, 2, \ldots , t_f-1$.

Now it is the private sector's turn either to accept or to reject this proposal, the first move leading to the agreement $\tilde{u}^1$ and the latter to disagreement controls in the current round: $(u_i(1) = u_i^d(1))$, $i = 1,2$ and to another proposal by the private sector in the third round $(t=2)$ $\tilde{u}^2 := (u(0), \ldots, u(t_f-1))$ with $u(0) = u^d(0)$, $u(1) = u^d(1)$ and $u(s) \in U^S$, $s = 2, 3, \ldots, t_f-1$. Etcetera.

A typical proposal in period t $(t = 0, 1, 2, \ldots, t_f)$ is:

$$\tilde{u}^t := (u(0), u(1), \ldots, u(t_f-1))$$

$$\text{with } u(s) = u^d(s) \text{ if } s < t$$
$$u(s) \in U^S \quad \text{if } s \geq t$$

Here $u^d(s) = (u_1^d(s), u_2^d(s))$ are the disagreement controls in previous rounds. For the remaining controls $(s \geq t)$ any $u(s) \in U^S$ may be chosen. Notice that $\tilde{u}^t$ completely characterizes the possible control outcomes of the policy game 3: any pair of strategies (f,g)

$$f := (A_0, \tilde{u}^1, A_2, \ldots, \tilde{u}^{t_f-1}) : \text{government's strategy}$$

$$g := (\tilde{u}^0, A_1, \tilde{u}^2, \ldots, A_{t_f-1}) : \text{private sector's strategy}$$

either leads to immediate agreement $(P(f,g) = \tilde{u}^0)$ or to some later agreement $(P(f,g) = \tilde{u}^t$, $t = 1, 2, \ldots, t_f-1)$ or to no agreement at all $(P(f,g) = \tilde{u}^{t_f})$[15].

5.2 Nash Equilibrium

An important assumption in Rubinstein's bargaining model is Pareto optimality of every proposal (i.c. wage is desirable for the worker and undesirable for the firm; in other words: no wage is preferred by both players; the players have conflicting objectives). This assumption is made in the policy game also: every controlproposal $\tilde{u}^t$ is required to be Pareto optimal/group-rational.

As in section 2 (proposition 1) and in section 4 (proposition 5) the game 3 has many Nash equilibria.

Proposition 9 (NE in policy game 3)
Let $\tilde{u}^t$ be Pareto optimal. Then:

$$(f^*, g^*) \text{ NE such that } P(f^*, g^*) = \tilde{u}^t$$

$$\Leftrightarrow \tilde{u}^t \succsim_1 \tilde{u}^{t_f} \wedge \tilde{u}^t \succsim_2 \tilde{u}^{t_f}$$

Proof
The arguments from the proof of proposition 1 apply. Clearly if one of the conditions does not hold, then $\tilde{u}^t$ cannot be a NE. If the conditions do hold, a NE is to propose that (Pareto optimal) proposal that pays best and to reject anything else until t is reached. In period t the controls $\tilde{u}^t$ are agreed upon. □

Under Pareto optimality, the set of NE outcomes coïncides with the set of individual-rational outcomes (i.e. the set of outcomes that both players prefer above the disagreement outcome/threat point $\tilde{u}^{t_f}$). As time goes by, the set of available controlvectors (and it's subset of Pareto-optimal and individual-rational controlvectors) shrinks, until in the end only the disagreement controlvector $\tilde{u}^{t_f}$ remains. This is an incentive for the decisionmakers to reach an agreement in an early stage of the game.
Formally, let ψ^t be the set of available controlvectors in period $t (t = 0, 1, 2, \ldots, t_f)$:

$$\psi^t := \{\tilde{u}^t | u(s) = u^d(s),\ s < t \wedge u(s) \in U^s,\ s \geq t\}$$

and let Ω^t be the Pareto-optimal and individualrational subset:

$$\Omega^t := \left[\tilde{u}^t \in \psi^t \middle| \left[\nexists \tilde{u}^t \in \psi^t : \tilde{u}^t >_i \tilde{u}^t,\ i = 1, 2\right] \wedge \right.$$

$$\left. \tilde{u}^t \succsim_i \tilde{u}^{t_f},\ i = 1, 2. \right]$$

Notice that $\Omega^{t_f} = \{\tilde{u}^{t_f}\}$.

Proposition 10 (Incentive for an early agreement)

Let $t_1 < t_2$; $t_1, t_2 \in \{0, 1, \dots, t_f\}$. Then:

$\forall \tilde{u}^{t_2} \in \Omega^{t_2} \quad \exists \tilde{u}^{t_1} \in \Omega^{t_1}$ such that $\tilde{u}^{t_1} \underset{\sim i}{>} \tilde{u}^{t_2}$, $i = 1, 2$.

Proof

By definition of ψ^t, $\psi^{t_2} \subset \psi^{t_1}$.

Choose a vector $\tilde{u}^{t_2} \in \Omega^{t_2}$ and consider the points in $\psi^{t_1} \backslash \psi^{t_2}$ that dominate $\tilde{u}^{t_2}$:

$$\Phi := \Phi^{t_1}(\tilde{u}^{t_2}) := \left\{ \tilde{v}^{t_1} \in \psi^{t_1} \backslash \psi^{t_2} \,\middle|\, \tilde{v}^{t_1} >_i \tilde{u}^{t_2},\ i = 1, 2 \right\}$$

If $\Phi = \emptyset$ then take $\tilde{u}^{t_1} = \tilde{u}^{t_2}$

If $\Phi \neq \emptyset$ then take any $\tilde{u}^{t_1}$ such that $\tilde{u}^{t_1} \underset{\sim i}{>} \tilde{v}^{t_1}$, $\quad \tilde{v}^{t_1} \in \Phi$.

By construction $\tilde{u}^{t_1} \in \Omega^{t_1}$ □

To illustrate proposition 10, consider a (utility-) function $J = (J_1, J_2)$ that represents the players' preferences:

$$J : \prod_{k=0}^{t_f-1} U^k \longrightarrow \mathbb{R}^2 \text{ such that for } \tilde{u}^t, \tilde{v}^s \in \prod_{k=0}^{t_f-1} U^k:$$

$$\tilde{u}^t \underset{\sim i}{>} \tilde{v}^s \Longleftrightarrow J_i(\tilde{u}^t) \geq J_i(\tilde{v}^s),\ i = 1, 2$$

The sets X^t, Y^t and Z correspond to ψ^t, Ω^t and Φ:

$$X^t := \{J(\tilde{u}^t) \in \mathbb{R}^2 | \tilde{u}^t \in \psi^t\} \subset \mathbb{R}^2$$

$$Y^t := \{J(\tilde{u}^t) \in X^t | \tilde{u}^t \in \Omega^t\} \subset X^t$$

$$Z(\hat{J}) := \left\{ J \in X^{t_1} \backslash X^{t_2} | J_i > \hat{J},\ i = 1, 2 \right\},$$

$$\hat{J} \in Y^{t_2}$$

This is the case Stefanski and Cichaki (1986) consider.

Figure 7

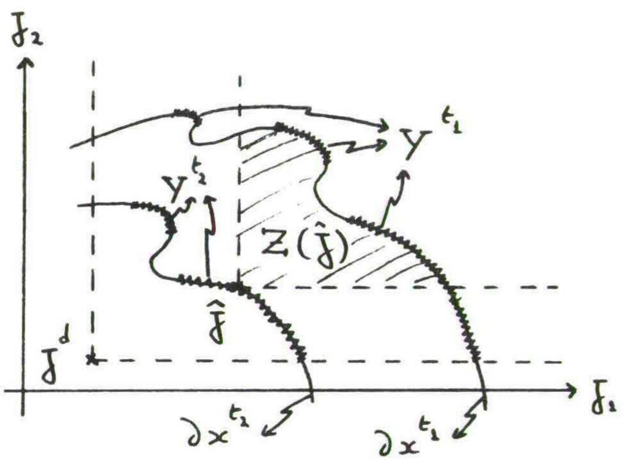


Illustration of proposition 10

$t_1 < t_2$: $\hat{J} \in T^{t_2}$

$\exists J \in Y^{t_1}$ such that

$J_1 \geq \hat{J}_1 \wedge J_2 \geq \hat{J}_2$.

Notice that from proposition 9 and the definition of Ω^t it follows that every $\tilde{u}^t \in \Omega^t$ can be a NE outcome of the policy game; in terms of figure 5: every pay-off $J = (J_1, J_2) \in Y^t$ is supported by some pair of NE strategies.

Proposition 10 indicates that this conclusion is not very satisfactiory. If subgame perfectness is introduced as an additional requirement (see section 2, definition 2) then PE bargaining strategies generally lead to a unique PE outcome; a proposition can be derived that resembles the propositions 2 and 4 of section 2 respectively 3 (proposition 11 below).

5.3 Perfect Equilibrium

To characterize PE strategies and PE outcomes in the game 3, define the following (multi-) functions:

$$d_1{}^t : \Omega^{t+1} \longrightarrow \Omega^t \qquad , \ t = 0,\ 2,\ \ldots\ ,\ t_f\text{-}2.$$

$$d_1{}^t(\tilde{u}^{t+1}) = \left\{\tilde{u}^t \in \Omega^t \,|\, \tilde{u}^t \succsim_1 \tilde{u}^{t+1} \wedge \forall \tilde{v}^t \in \Omega^t\colon \tilde{u}^t \succsim_2 \tilde{v}^t\right\}$$

If the players can be sure an outcome $\tilde{u}^{t+1}$ will be reached at t + 1, then the private sector (player 2) shall propose a control $\tilde{u}^t$ in the set $d_1{}^t(\tilde{u}^{t+1})$; thereby it maximizes it's own utility while knowing the government has nothing better to do than to accept $\tilde{u}^t (\tilde{u}^t \succsim_1 \tilde{u}^{t+1})$. Similarly, if an outcome $\tilde{u}^{t+2}$ is expected in the next round, then the government shall propose a $\tilde{u}^{t+1}$ in the set $d_2{}^{t+1}(\tilde{u}^{t+2})$:

$$d_2{}^{t+1}\colon \Omega^{t+2} \longrightarrow \Omega^{t+1} \qquad , \ t = 0,\ 2,\ \ldots\ ,\ t_f\text{-}2$$

$$d_2{}^{t+1}(\tilde{u}^{t+2}) = \left\{\tilde{u}^{t+1} \in \Omega^{t+1} \,|\, \tilde{u}^{t+1} \succsim_2 \tilde{u}^{t+2} \wedge \quad \tilde{v}^{t+1} \in \Omega^{t+1}\colon \tilde{u}^{t+1} \succsim_1 \tilde{v}^{t+1}\right\}$$

Now assume Ω^t, Ω^{t+1} are compact and preferences $\succsim_i$, i = 1, 2 are continuous. Then $d_1{}^t$ and $d_2{}^t$ are not empty and PE strategies and PE outcomes can be computed as follows:

<u>Proposition 11</u> (PE in game 3)

A) $\tilde{y}^t \in \Omega^t$ is a PE control proposal by the private sector in round t and $\tilde{x}^{t+1} \in \Omega^{t+1}$ is a PE control proposal by the government in round t + 1,

$t \in \{0, 2, 4, \ldots, t_f-2\} \Leftrightarrow$

$\tilde{y}^0 \in d_1{}^0(\tilde{x}^1)$; $\tilde{x}^1 \in d_2{}^1(\tilde{y}^2)$;

$\tilde{y}^2 \in d_1{}^2(\tilde{x}^3)$; $\tilde{x}^3 \in d_2{}^3(\tilde{y}^4)$;

$\vdots$

$\tilde{y}^{t_f-2} \in d_1{}^{t_f-2}(\tilde{x}^{t_f-1})$; $\tilde{x}^{t_f-1} \in d_2{}^{t_f-1}(\tilde{y}^{t_f})$

with $\tilde{y}^{t_f} := \tilde{u}^{t_f}$: disagreement controls.

B) At least one sequence of proposals $\tilde{y}^t$, $\hat{x}^{t+1}$ $t = 0, 2, \ldots, t_f-2$ as described above exists.

C) $\tilde{u}^t$, $\tilde{u}^{t+1}$, $t = 0,2,\ldots,t_f-2$ is a PE outcome of the subgame starting in round t resp. t+1
$\Leftrightarrow \tilde{u}^t = \tilde{y}^t$ resp. $\tilde{u}^{t+1} = \tilde{x}^{t+1}$.

Proof

The proof is similar to the proof of the propositions 2 and 4. PE strategies are:

$$f^* = (A_0, \tilde{x}^1, \ldots, A_{t_f-2}, \tilde{x}^{t_f-1})$$

$$g^* = (\tilde{y}^0, A_1, \ldots, \tilde{y}^{t_f-2}, A_{t_f-1})$$

with

$$A_t = Y \text{ if } \tilde{u}^t \gtrsim_1 \tilde{y}^t$$

$$= N \text{ if } \tilde{u}^t <_1 \tilde{y}^t$$

$$A_{t+1} = Y \text{ if } \tilde{u}^{t+1} \underset{\sim}{>}_2 \tilde{x}^{t+1}$$

$$= N \text{ if } \tilde{u}^{t+1} <_2 \tilde{x}^{t+1}, \; t = 0, 2, \ldots, t_f-2.$$

□

<u>Example</u> (cf. de Zeeuw (1984), example 4.3.1)
Consider the policy game 3, figure 5 with $t_f = 2$ bargaining rounds. In the first round (t=0), the private sector (player 2) proposes $\tilde{u}^0 = (u(0), u(1))$ with $u(0) = (u_1(0), u_2(0)) \in U^0$ and $u(1) = (u_1(1), u_2(1)) \in U^1$. If the government does not agree it can make a counterproposal $\tilde{u}^1 = (u(0), u(1))$ with $u(0) = (u_{10}, u_{20})$: disagreement controls in first round and $u(1) = (u_1(1), u_2(1)) \in U^1$. If on turn the private sector rejects $\tilde{u}^1$, then total disagreement results:

$$\tilde{u}^2 = (u(0), u(1))$$

with

$$u(0) = (u_{10}, u_{20})$$

$$u(1) = (u_{11}, u_{21}).$$

Let the system's dynamics be:

$$x(0) = x_0$$

$$x(1) = x(0) + u_1(0) + u_2(0)$$

$$x(2) = x(1) + u_1(1) + u_2(1)$$

and let $J = (J_1, J_2)$ represent the players' preferences:

$$J_1(\tilde{u}^t) = -\tfrac{1}{2}\left[x^2(0) + u_1^{\,2}(0) + x^2(1) + u_1^{\,2}(1) + x^2(2)\right]$$

$$J_2(\tilde{u}^t) = -\tfrac{1}{2}\left[2x^2(0) + u_2^{\,2}(0) + 2x^2(1) + u_2^{\,2}(1) + 2x^2(2)\right]$$

To find a PE outcome $\tilde{y}^0$, first a PE outcome $\tilde{x}^1$ of the subgame starting at $t = 1$ has to be computed. Applying proposition 11 gives:

$$\tilde{x}^1 \in d_2^{\,1}(\tilde{u}^2) \Leftrightarrow \tilde{x}^1 = \arg\max_{\tilde{u}^1} J_1(\tilde{u}^1) \text{ s.t. } J_2(\tilde{u}^1) \geq J_2(\tilde{u}^2)$$

The second step is to find $\tilde{y}^0 \in d_1^{\,0}(\tilde{x}^1)$:

$$\tilde{y}^0 \in d_1^{\,0}(\tilde{x}^1) \Leftrightarrow \tilde{y}^0 = \arg\max_{\tilde{u}^0} J_2(\tilde{u}^0) \text{ s.t. } J_1(\tilde{u}^0) \geq J_1(\tilde{x}^1)$$

Clearly, the equilibrium depends heavily on the disagreement controls $\tilde{u}^2$.

6 Summary and Conclusion

Traditionally bargaining theory has focussed on 'general properties that "any reasonable solution" should possess' (quotation Nash (1953), p. 136). The Nash equilibrium (NE) axioms (see Nash (1950)) form the most famous set of properties leading to one single outcome (the solution) of the typical static two-person bargaining problem[16]. Modifications have been proposed by Kalai and Smorondinsky (1975) and Binmore (1984).
Such an axiomatic approach is attractive because of it's generality. But at the same time this means that it cannot take into account the role of dynamic features of specific bargaining situations. This motivated Nash (1953) to describe his static bargaining problem as a two-stage (extensive) game; in the first stage disagreement strategies are announced (the threats) and in the second stage the (simultaneous) demands are matched. Following such a strategic approach (which concentrates on general properties that any reasonable strategy pair should possess) and applying the basic Nash equilibrium (NE) concept (Nash (1951)), he found that in this case dynamic NE strategies lead to the same solution as was singled out by the static NE axioms. This strongly advocated the axiomatic approach.

However, if commitment is not possible, - which is typically the case in most bargaining situations -, then the NE concept is not consistent, i.e. it may involve threats for which it is not rational to execute them if an opponent deviates from the equilibrium.
To rule out the possibility of such incredible threats Selten (1975) refined the NE concept and since Rubinstein (1982) implemented Selten's perfect equilibrium (PE) concept into a simple but very natural bargaining framework, the strategic approach has received much renewed attention in bargaining theory.

Starting from Rubinstein's seminal model, several aspects of strategic bargaining have been addressed in this paper. It is interesting to see how procedures and outside options can affect the equilibrium moves and outcomes in the bargaining game (section 2 and 3). If information is incomplete, the transmission of information (i.c. the way a reputation evolves)

plays a crucial role in deriving equilibrium; moreover, in this case disagreement can be explained (section 4).
Finally it is pointed out that the basic model can also be used to analyse bargaining problems with other pay-off structures, such as policy games (section 5). Another extension in this direction could be the introduction of employment level as an additional goal (besides wage level) for the union.

In addition to this, three topics seem important for further research.
Firstly, there is the question of how outcomes of the strategic approach relate to outcomes of the axiomatic approach. For example in Binmore, Rubinstein and Wolinsky (1985) it is shown that some Rubinstein-type models yield precisely the axiomatic asymmetric NE outcome in the limit (as the length of bargaining rounds tend to zero). Secondly, to explain disagreement, further analysis of models with incomplete information is needed; the requirements that should be made on the transmission of information is a central issue (see Rubinstein (1985b) for an overview of options; see also Kreps and Wilson (1982b) and Barro and Gordon (1983) for applications in related area). Thirdly, the extension to three (or more) players is a further challenge (see e.g. Binmore (1985), Haller (1986)).

Notes

1. Since the insider-outsider model of section 3 can be seen as a generalisation of the basic model, the effect of different reaction times in the basic model follows as a special case. See Binmore, Rubinstein and Wolinsky (1985) for the analysis of several effects in the basic model.

2. Originally perfect equilibrium (PE) was defined for extensive form games with complete information. The generalisation to incomplete information is given in Kreps and Wilson (1982a) and has become known as sequential equilibrium (SE). See further section 4.

3. See section 6 for a discussion of different approaches.

4. These assumptions are maintained throughout the paper.

5. A discount<u>factor</u>, δ_i, is inversely related to the corresponding discount<u>rate</u>, say r_i. For $r_i \in [0,\infty)$ one can write $\delta_i = e^{-r_i}$, thus obtaining $\delta_i \in (0,1]$.

6. These assumptions also are maintained throughout the paper.

7. The importance of such utility-functions is emphasized in Fishburn and Rubinstein (1982).
   They show that preferences that satisfy the assumptions made thus far (i.e. time is valuable for both, wage is desirable for the worker and profit for the firm, continuity, stationarity and increasing compensation) can be represented by utility-functions of the form $u_i(w,t) = \beta_i^t\ v_i(w)$, for some $\beta_i \in [0,1]$ and some continuous functions v_i, with v_1 increasing and v_2 decreasing; $i = 1,2$.
   For illustrative purposes $v_1(w) = w$ and $v_2(w) = 1-w$ are chosen.

8. To define formally risk aversion within a bargaining round, consider some wage y which is preferred by player i above some other wage x. Let $p. x \oplus (1-p).y$ denote the lottery over x and y with probability p and $(1-p)$ respectively, $p \in (0,1)$. Let further $z: = p x + (1-p)y$ be the expected wage if the lottery is performed. Player i is said to be risk averse (resp. risk neutral resp. risk loving) on the interval $[a,b]$ if

$$\forall x, y \in [a,b] \quad \forall p \in (0,1):$$

$$y \succ_i x \Rightarrow z \succ_i p. x \oplus (1-p). y$$

$$(\text{resp. } z \sim_i p. x \oplus (1-p). y ,$$

$$\text{resp. } z \prec_i p. x \oplus (1-p). y)$$

If the utility function satisfies the socalled expected utility property $u_i (z) = p\, u(x) + (1-p)\, u(y)$ (see Von Neumann and Morgenstern (1944)), then risk aversion is precisely a concave transformation over the utilities.

9. That is: $1 - y_o = \dfrac{1-\delta^T}{(1+\delta)(1-\delta^{T-1})}$ is the unique PE profit that the firm shall propose (Shaked and Sutton (1984), proposition).

10. This corresponding problem is to choose a point of utilities $u=(u_1, u_2)$ out of the possibility-set S, given a threat-point $d=(d_1, d_2)$. The Nash axioms (Nash (1950)) then lead to the unique solution
$u^* = \arg\max_{u \in S} (u_1-d_1)(u_2-d_2)$.
Currently $S: = \{(u_1, u_2) \mid u_1 + u_2 \leq 1\}$
and $d: = (0,0)$ (i.e. perpetual disagreement). Thus $u^* = (\frac{1}{2}, \frac{1}{2})$.

11. In appendix A these conclusions are derived formally.

12. To see this consider two possible events:

E_1 : the firm is weak

$E_2(t)$: round t is reached, $t = 0,2,4,\ldots$

(i.e. the firm has rejected w_{t-1} and proposes w_t)

According to Bayes' rule the worker's belief that the firm is weak must satisfy:

$$p(t) := \Pr\{E_1 | E_2(t)\} = \Pr\{E_1 \cap E_2(t)\} / \Pr\{E_2(t)\},$$
$$t = 0,2,4,\ldots \text{ and } \Pr\{E_2(t)\} > 0.$$

Thus if $\underline{A}_{t-1} = N \wedge \underline{w}_t = w_t \wedge [\bar{A}_{t-1} = Y \vee \bar{w}_t \neq w_t]$ then $\Pr\{E_1 \cap E_2\ (t)\} = \Pr\{E_2\ (t)\}$ and $p(t) = 1$.

Also if $\bar{A}_{t-1} = N \wedge \bar{w}_t = w_t \wedge [\underline{A}_{t-1} = Y \vee \underline{w}_t \neq w_t]$ then $\Pr\{E_1 \cap E_2(t)\} = 0$ and $p(t) = 0$.

Finally, if $\underline{A}_{t-1} = \bar{A}_{t-1} = N \wedge \underline{w}_t = \bar{w}_t = w_t$ then $\Pr\{E_1 \cap E_2(t)\} = \Pr\{E_1\}.\ \Pr\{E_2(t)\}$ and $p(t) = p(t-2)$.

13. See appendix B for the derivation of this result.

14. In a more general set up bargaining will concentrate on the shape of the control functions G_i (i.e. on <u>how</u> future actions should be undertaken, given the state reached) rather than on the value of the controlvariables u_i themselves (i.e. on <u>what</u> specific future actions should be undertaken). The role of uncertainty, as reflected in the appearance of time as an explicit argument in the functions F and G_i, can then be taken into account.
This generalisation is left for further research.

15. An agreement is binding, i.e. once it is reached, none of the players can deviate from the controls agreed upon. An other option is to consider (non-binding) announcements rather than (binding) agreements. Deviation from announcements on future actions is then allowed for, but may lead to a loss of reputation (see Barro and Gordon (1983)). Incomplete information has then entered the game and in describing equilibrium a crucial role is played by the way in which the reputation is assumed to evolve along all possible trajectories of the game (see section 4).

16. See note 11.

17. Willy Spanjers helped to overcome some mathematical problems in the appendices.

Appendix[17] A

Let $x = \delta_1$ and $y = \delta_2$ be the discountfactors of the worker and the firm respectively.
The qualitative effect of changing discountfactors can be summarised as follows:

Lemma
Let $k \in \mathbb{N}$ and consider the function $F_k : I \times I \to I$, $I := [0,1]$,

$$F_k(x,y) := \frac{x(1-y)}{1-xy} \; \frac{1-x^k y^k}{1-x^{k+1} y^k}, \quad x\, y \neq 1.$$

Then $\frac{\partial F_k}{\partial x} \geq 0$ and $\frac{\partial F_k}{\partial y} \leq 0$, $\forall\, k \in \mathbb{N}$.

Proof
For $k = 0$, $\frac{\partial F_k}{\partial x} = \frac{\partial F_k}{\partial y} = 0$ because $F_k(x,y) = 0$.

Let $f(x,y) := \frac{x(1-y)}{1-xy}$ and $g_k(x,y) := \frac{1-x^k y^k}{1-x^{k+1} y^k}$

Then $F_k = f * g_k$ and $\frac{\partial F_k}{\partial z} = \frac{\partial f}{\partial z} \cdot g_k + \frac{\partial g_k}{\partial z} \cdot f$, $z = x \vee y$.

$$\frac{\partial f}{\partial x} = \frac{1-y}{(1-xy)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x\,(x-1)}{(1-xy)^2}$$

$$\frac{\partial g_k}{\partial x} = \frac{k\, y^k\,(x^k - x^{k-1}) + x^k y^k (1-x^k\, y^k)}{(1-x^{k+1}\, y^k)^2}$$

$$\frac{\partial g_k}{\partial y} = \frac{k\, y^{k-1}\,(x^{k+1} - x^k)}{(1-x^{k+1}\, y^k)^2}$$

Since $f \geq 0$, $g_k > 0$, $\frac{\partial f}{\partial y} \leq 0$ and $\frac{\partial g_k}{\partial y} \leq 0$ the result for y follows immediately: $\frac{\partial F_k}{\partial y} \leq 0$

For x we can write:

$$(1-xy)^2.(1-x^{k+1}\ y^k)^2.\ \frac{\partial\ F_k}{\partial\ x} = (1-x^k\ y^k).(1-x^{k+2}\ y^{k+1}) +$$

$$(1-x\ y).\ k.\ x^k\ y^k\ (x-1)$$

Thus:

$$\frac{\partial\ F_k}{\partial\ x} \geq 0 \Leftrightarrow (1-x^k\ y^k)(1-x^{k+2}\ y^{k+1}) + (1-x\ y)\ k\ x^k\ y^k\ (x-1).\ \geq 0.$$

For $k \geq 1$, $x\ y \geq x^k\ y^k$.
It is therefore sufficient tot show that:

$$1-x^{k+2}\ y^{k+1} + k\ x^k\ y^k\ (x-1) \geq 0.$$

Finally, $- x^{k+1}\ y^{k+1} \leq - x^{k+2}\ y^{k+1}$
and

$$x^{k+1}\ y^k \leq x^{k+1}\ y^{k+1}$$

and thus it is sufficient to show that:

$$1 - (xy)^{k+1} + k(xy)^{k+1} - k(xy)^k \geq 0.$$

The function $h_k(v) := 1 - v^{k+1} + k\ v^{k+1} - k\ v^k$, $v := xy \in [0,1)$ is decreasing, $\forall\ k = 1,2,\ldots$ and $\lim_{v \uparrow 1} h_k(v) = 0$.

Thus $h_k(v) \geq 0$ and $\frac{\partial\ F_k}{\partial\ x} \geq 0$

□

Appendix B

Lemma

$\forall\ \delta \in (0,1)\quad \forall\ \alpha,\ \beta$ such that $0 \le \alpha < \beta \le 1$:

$$\left[1-\alpha + \alpha\,\frac{\delta(1-\beta)}{1-\beta\ \delta} \le 1 - \beta + \beta\,\frac{\delta(1-\alpha)}{1-\alpha\ \delta} \Leftrightarrow (\alpha + \beta)\ \delta \ge 1\right]$$

Proof

Let $\delta \in (0,1)$ and $0 \le \alpha < \beta \le 1$.

Then

$$\beta - \alpha - \beta\,\frac{\delta(1-\alpha)}{1-\alpha\ \delta} + \alpha\,\frac{\delta(1-\beta)}{1-\beta\ \delta} \le 0$$

$$\Leftrightarrow (\beta-\alpha)(1-\alpha\delta)(1-\beta\delta) - \beta\ \delta\ (1-\alpha)(1-\beta\delta) + \alpha\ \delta\ (1-\beta)(1-\alpha\delta) \le 0$$

$$\Leftrightarrow \beta - \beta^2\ \delta - \alpha + \alpha^2\ \delta - \beta\ \delta + \beta^2\ \delta^2 + \alpha\ \delta - \alpha^2\ \delta^2 \le 0$$

$$\Leftrightarrow (\beta-\alpha)(1-\delta)(1 - (\alpha+\beta)\ \delta) \le 0$$

$$\Leftrightarrow (\alpha+\beta)\ \delta \ge 1$$

$\square$

References

Barro, R. and D. Gordon (1983): "Rules, Discretion and Reputation in a Model of Monetary Policy". Journal of Monetary economics, 12, pp. 101-121.

Basar, T. and G. Olsder (1982): Dynamic Noncooperative Game Theory. Academic Press, New York.

Binmore, K. (1984): "Bargaining Conventions". International Journal of Game Theory, 13, pp. 193-200.

Binmore, K. (1985): "Bargaining and Coalitions". In: Game Theoretic Models of Bargaining; A. Roth (ed.), Cambridge University Press, pp. 269-304.

Binmore, K. and M. Herrero (1985): "Matching and Bargaining in Dynamic Markets". Discussionpaper ICERD, London School of Economics.

Binmore, K., A. Rubinstein and A. Wolinsky (1985): "The Nash Bargaining Solution in Economic Modelling". Discussionpaper ICERD, London School of Economics.

Fishburn, P. and A. Rubinstein (1982): "Time Preferences". International Economic Review, 23, pp. 677-694.

Fudenberg, D. and J. Tirole (1983): "Sequential Bargaining under Incomplete Information". Review of Economic Studies, 50, pp. 221-248.

Grossman, G. and J. Richardson (1985): "Strategic Trade Policy; a Survey of Issues and Early Analysis". Special papers in international economics, 15, Princeton University.

Haller, H. (1986): "Non-cooperative Bargaining of $N \geq 3$ Players". Economic Letters, 22, pp. 11-13.

Harsanyi, J. (1967/1968): "Games with Incomplete Information Played by 'Bayesian' Players; part I, II and III". Management Science, 14, pp. 159-182, 320-334, 486-502.

Kalai, E. and M. Smorodinsky (1975): "Other Solutions to Nash's Bargaining Problem". Econometrica, 43, pp. 513-518.

Kooreman, P. and A. Kapteyn (1985): "Estimation of a Game-theoretic Model of Household Labor Supply". Discussionpaper FEW, Tilburg University.

Kreps, D. and R. Wilson (1982a): "Sequential Equilibria" Econometrica, 50, pp. 863-894.

Kreps, D. and R. Wilson (1982b): "Reputation and Imperfect Information". Journal of Economic theory, 27, pp. 253-279.

Meijdam, A. and A. de Zeeuw (1986): "On Expectations Information and Dynamic Game Equilibria". In: Dynamic Games and Applications in Economics, T. Basar (ed.), Lecture Notes in Economics and Mathematical Systems, 265, pp. 26-46,.

Nash, J. (1950): "The Bargaining Problem". Econometrica, 18, pp. 155-162.

Nash, J. (1951): "Noncooperative Games". Annals of Mathematics, 54, pp. 286-295.

Nash, J. (1953): "Two-person Cooperative Games". Econometrica, 21, pp. 128-140.

Von Neumann, J. and O. Morgenstern (1944): Theory of Games and Economic Behavior. Princeton University Press.

Perry, M. (1986): "An Example of Price Formation in Bilateral Situations: a Bargaining Model with Incomplete Information". Econometrica, 54, pp. 313-321.

Roth, A. (1985): "A Note on Risk Aversion in a Perfect Equilibrium Model of Bargaining". Econometrica, 53, pp. 207-211.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model". Econometrica, 50, pp. 97-109.

Rubinstein, A. (1985a): "A Bargaining Model with Incomplete Information about Time Preferences". Econometrica, 53, pp. 1151-1172.

Rubinstein, A. (1985b): "Choice of Conjectures in a Bargaining Game with Incomplete Information". In: Game Theoretic Models of Bargaining; A. Roth (ed.) Cambridge University Press, pp. 99-114.

Rubinstein, A. and A. Wolinsky (1985): "Equilibrium in a Market with Sequential Bargaining". Econometrica, 53, pp. 1133-1150.

Selten, R. (1975): "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games". International Journal of Game Theory, 4, pp. 25-55.

Shaked, A. and J. Sutton (1984): "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model". Econometrica, 52, pp. 1351-1364.

Stefanski, J. and K. Cichocki (1986): "Strategic Bargaining in a Dynamic Economy". Discussionpaper Systems Research Institute, Polish Academy of Science.

Sutton, J. (1986): "Non-Cooperative Bargaining Theory: An Introduction". Review of Economic Studies, 8, pp. 709-724.

Withagen, C. (1984): Economic Theory and International Trade in Natural Exhaustible Resources. Ph. D. Thesis Tilburg University.

De Zeeuw, A. (1984): Difference Games and Linked Econometric Policy Models. Ph. D. Thesis, Tilburg University.

IN 1986 REEDS VERSCHENEN

202 J.H.F. Schilderinck
Interregional Structure of the European Community. Part III

203 Antoon van den Elzen and Dolf Talman
A new strategy-adjustment process for computing a Nash equilibrium in a noncooperative more-person game

204 Jan Vingerhoets
Fabrication of copper and copper semis in developing countries. A review of evidence and opportunities

205 R. Heuts, J. van Lieshout, K. Baken
An inventory model: what is the influence of the shape of the lead time demand distribution?

206 A. van Soest, P. Kooreman
A Microeconometric Analysis of Vacation Behavior

207 F. Boekema, A. Nagelkerke
Labour Relations, Networks, Job-creation and Regional Development. A view to the consequences of technological change

208 R. Alessie, A. Kapteyn
Habit Formation and Interdependent Preferences in the Almost Ideal Demand System

209 T. Wansbeek, A. Kapteyn
Estimation of the error components model with incomplete panels

210 A.L. Hempenius
The relation between dividends and profits

211 J. Kriens, J.Th. van Lieshout
A generalisation and some properties of Markowitz' portfolio selection method

212 Jack P.C. Kleijnen and Charles R. Standridge
Experimental design and regression analysis in simulation: an FMS case study

213 T.M. Doup, A.H. van den Elzen and A.J.J. Talman
Simplicial algorithms for solving the non-linear complementarity problem on the simplotope

214 A.J.W. van de Gevel
The theory of wage differentials: a correction

215 J.P.C. Kleijnen, W. van Groenendaal
Regression analysis of factorial designs with sequential replication

216 T.E. Nijman and F.C. Palm
Consistent estimation of rational expectations models

217 P.M. Kort
The firm's investment policy under a concave adjustment cost function

218 J.P.C. Kleijnen
Decision Support Systems (DSS), en de kleren van de keizer ...

219 T.M. Doup and A.J.J. Talman
A continuous deformation algorithm on the product space of unit simplices

220 T.M. Doup and A.J.J. Talman
The 2-ray algorithm for solving equilibrium problems on the unit simplex

221 Th. van de Klundert, P. Peters
Price Inertia in a Macroeconomic Model of Monopolistic Competition

222 Christian Mulder
Testing Korteweg's rational expectations model for a small open economy

223 A.C. Meijdam, J.E.J. Plasmans
Maximum Likelihood Estimation of Econometric Models with Rational Expectations of Current Endogenous Variables

224 Arie Kapteyn, Peter Kooreman, Arthur van Soest
Non-convex budget sets, institutional constraints and imposition of concavity in a flexible household labor supply model

225 R.J. de Groof
Internationale coördinatie van economische politiek in een twee-regio-twee-sectoren model

226 Arthur van Soest, Peter Kooreman
Comment on 'Microeconometric Demand Systems with Binding Non-Negativity Constraints: The Dual Approach'

227 A.J.J. Talman and Y. Yamamoto
A globally convergent simplicial algorithm for stationary point problems on polytopes

228 Jack P.C. Kleijnen, Peter C.A. Karremans, Wim K. Oortwijn, Willem J.H. van Groenendaal
Jackknifing estimated weighted least squares

229 A.H. van den Elzen and G. van der Laan
A price adjustment for an economy with a block-diagonal pattern

230 M.H.C. Paardekooper
Jacobi-type algorithms for eigenvalues on vector- and parallel computer

231 J.P.C. Kleijnen
Analyzing simulation experiments with common random numbers

232 A.B.T.M. van Schaik, R.J. Mulder
On Superimposed Recurrent Cycles

233 M.H.C. Paardekooper
Sameh's parallel eigenvalue algorithm revisited

234 Pieter H.M. Ruys and Ton J.A. Storcken
Preferences revealed by the choice of friends

235 C.J.J. Huys en E.N. Kertzman
Effectieve belastingtarieven en kapitaalkosten

236 A.M.H. Gerards
An extension of König's theorem to graphs with no odd-K_4

237 A.M.H. Gerards and A. Schrijver
Signed Graphs - Regular Matroids - Grafts

238 Rob J.M. Alessie and Arie Kapteyn
Consumption, Savings and Demography

239 A.J. van Reeken
Begrippen rondom "kwaliteit"

240 Th.E. Nijman and F.C. Palmer
Efficiency gains due to using missing data. Procedures in regression models

241 Dr. S.C.W. Eijffinger
The determinants of the currencies within the European Monetary System

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen

254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint products: A comment

255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?

256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts